Applied Mechanics Reviews

A Critical Review of the World Literature in Applied Mechanics

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Published by The American Society of Mechanical Engineers

October 1949 **Vol. 2, No. 10**

Applied Mechanics Reviews

Published Monthly by The American Society of Mechanical Engineers
at Easton, Pa., with the co-operation of

THE OFFICE OF NAVAL RESEARCH · ILLINOIS INSTITUTE OF TECHNOLOGY · THE ENGINEERING FOUNDATION
AMERICAN SOCIETY OF CIVIL ENGINEERS · AMERICAN INSTITUTE OF PHYSICS
INSTITUTE OF THE AERONAUTICAL SCIENCES · AMERICAN MATHEMATICAL SOCIETY
SOCIETY FOR EXPERIMENTAL STRESS ANALYSIS · THE INSTITUTION OF MECHANICAL ENGINEERS
THE ENGINEERING INSTITUTE OF CANADA

Editorial Office: APPLIED MECHANICS REVIEWS, Illinois Institute of Technology, Chicago 16, Ill., USA.
Subscription and Production Office: The American Society of Mechanical Engineers, 29 West 39th St., New York 18, N. Y., USA.

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Abbreviations of units follow the standard of Abbreviations for Scientific and Engineering Terms of the Am. Standards Assoc. Examples: psi (pounds per square inch); cps (cycles per second); mph (miles per hour).

APPLIED MECHANICS REVIEWS, October 1949, Vol. 2, No. 10. Published monthly by The American Society of Mechanical Engineers at 20th and Northampton Streets, Easton, Pa., USA. The editorial office is located at the Illinois Institute of Technology, Chicago 16, Ill., USA. Headquarters of ASME, 29 West 39th St., New York 18, N. Y., USA. Cable address "Dynamic," New York. Price $1.50 per copy, $12.50 a year; to members of ASME and co-operating societies $0.75 per copy, $9 a year. Changes of address must be received at Society headquarters three weeks before they are to be effective on the mailing list. Please send old as well as new address. . . . By-Laws: The Society shall not be responsible for statements or opinions advanced in papers or printed in its publications (B13, Par. 4). . . . Entered as second-class matter, January 11, 1948, at the Post Office at Easton, Pa., under the Act of March 3, 1897. . . .

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Communications and Queries

Concerning "Query about sprays," August 1949, p. 169.

Editor:

Refer A. M. Kuethe to "The production of sprays and mists of uniform drop size by means of spinning disk type sprayers," by Walton and Prewett, *Proceedings of the Physical Society London*, June 1949, vol. 62, sec. B, pp. 341–350.

Henry W. Parker,
Sylvania Electric Products, Inc., Flushing, N. Y.

Theoretical and Experimental Methods

(See also Revs. 1236, 1237, 1238, 1241, 1242, 1247, 1338)

1220. **V. J. Smirnov, "A course in higher mathematics, vol. 5"** (in Russian), Ogiz–Gostehhizdat, 1947. Paperboard, 8.8 × 6 in., 584 pp., 3 figs.

This fifth volume of an encyclopedic course of higher analysis covers Stieltjes integrals, set functions, measure theory, measurable functions, Lebesgue integrals, absolute continuity, generalized integrals, Hilbert spaces, bounded and unbounded operators.

Ed.

1221. **T. M. MacRobert, "Spherical harmonics,"** Dover Publications, Inc., New York, 1947. Cloth, 5.6 × 8.7 in., 372 pp., $4.50.

This second edition of a well-known treatise on spherical harmonics differs from the first (1928) only in the addition of two new chapters. The book opens with a few remarks on Fourier series, followed by treatments of heat conduction and of the transverse vibrations of a string. After a sketch of the properties of the hypergeometric function, there follows a detailed exposition of the properties of the Legendre polynomial, the Legendre functions, and the associated Legendre functions of integral order. Next are given numerous applications to the formal solution of problems in potential theory.

The first edition closed with a treatment of Bessel functions. In the new edition, the added chapters concern the hypergeometric function and the associated Legendre functions of arbitrary real order; it is unfortunate that the author did not see fit to introduce his new material on the hypergeometric function (13 pages) into chapter IV, where that function has already been treated, nor to integrate his material on the associated Legendre functions, for the present arrangement necessitates a rather tedious duplication of proofs. Misprints from the first edition, such as the transposition of lines on p. 164, have not been corrected. The functions $Q_n^m(X)$ and $T_n^m(X)$ are defined in this edition as $(-1)^m$ times the functions so denoted in the first edition.

An interesting example of the change in attitude toward applied mathematics in 20 years may be drawn from G. E. Raynor's remark [*Bull. Am. Math. Soc.*, 1928, vol. 34, pp. 779–780] in reviewing the first edition: "... (the author) is careful to keep before the reader the validity of the formulas as actually derived in the text. This is a very welcome quality in the book, a quality which is so often lacking in texts designed to appeal to the applied mathematician." In point of fact, the author's treatment of Fourier series is so meager that it does not even permit one to differentiate, let alone establish the existence of the simplest solution of a heat-conduction problem. On pp. 30–31 there is a "proof" of the uniqueness of solution of heat-flow boundary problems without any mention of the rather restrictive conditions under which this uniqueness holds. Recent American books on applied mathematics are much more helpful in this respect. Neither does the book represent the viewpoint of a physicist, for, with the exception of a most interesting discussion of the harp, violin, and piano strings in chapter III, no mechanical or physical interpretation of the solutions of the various boundary problems is presented.

It cannot be denied that the book is a useful collection of formulas and of methods of solving problems in potential theory. The avowed purpose, "a text-book on ... spherical harmonics ... without employing the method of contour integration," is difficult to appreciate, however. Surely, any person whose manipulative skill is sufficient to enable him to follow the author's elaborate formulas will wish to learn to use contour integration. But indeed the book reflects an abhorrence of general methods. Not even the simplest aspects of the general theory of orthogonal functions are presented. The orthogonality of the Legendre and Bessel functions is proved separately, and even the word orthogonality is not mentioned. It would be wrong to give the student the idea that general methods can do all that needs to be done with special functions, but to the reviewer it seems a pity to conceal the existence of these methods. In fact, a knowledge of the Fuchsian theory, orthogonal functions, contour integration, transform theory, and differential-difference equations is much more valuable to the applied mathematician or theoretical physicist than are the detailed properties of the Legendre and Bessel functions. It is more interesting, more stimulating, and more instructive to see what remains to be done with a given special function *after* all the general theories and methods have been applied to it. Perhaps also a sign of this avoidance of generality is the author's custom of filling a page with the first few terms of a series rather than writing down the general term. It is unfortunate also that usually no credit is given to the discoverers of the results presented. The foregoing remarks are not criticism of this carefully written book, full of useful detail, but rather an evidence of the change in the needs of applied mathematics since the first edition appeared.

C. A. Truesdell, USA

1222. **Herbert Buchholz, "Comments on an expansion formula in the theory of the Bessel function"** (in German), *Z. angew. Math. Mech.*, Nov.-Dec. 1947, vol. 25/27, pp. 245–252.

The difference between two products of Bessel and Hankel functions of the same order, suitably multiplied, is a one-valued function with simple poles at the zeros of J_ν. The corresponding development seems to be wrong in the treatise of Watson (p. 499) and in the Magnus-Oberhettinger collection of formulas, although correctly derived in the earlier works of Sommerfeld and of Carslaw. The author introduces three similar product differences, the second one containing first derivatives of the J and H functions, the third and fourth using different multiplying func-

tions. The second and fourth functions have simple poles at the zeros of J_ν'. The author derives the simple-fraction developments of all four functions, three of them being new. As a corollary he obtains the sums of several numerical series containing the zeros of J_ν', similar to those containing the zeros of J_ν, to be found in Watson and in Nielsen. The four functions here considered had been introduced by the author in 1941 in a study of radiation from cylindrical conductors. (In this paper the reader should watch for several missing differentiation signs.)

P. Le Corbeiller, USA

1223. H. Wittich, "On conformal mapping of simply connected regions" (in German), *Z. angew. Math. Mech.*, Aug.-Sept. 1947, vol. 25/27, pp. 131–132.

The author investigates methods for the conformal mapping of a simply connected region on the unit circle with respect to their suitability for practical computation. In aerodynamics the behavior of the mapping function in the neighborhood of the boundary is of special interest. A criterion of convergence and a numerical upper bound are given for the well-known method of Theodorsen and Garrick. Actual computations seem to indicate that this bound can be further increased. The criterion requires that the boundary of the given region differ little from the circle.

Manfred Schaefer, Germany

1224. L. Collatz, "Eigenvalue problems for certain linear integro-differential equations" (in German), *Z. angew. Math. Mech.*, Aug.-Sept. 1947, vol. 25/27, pp. 129–131.

A linear integrodifferential equation of the form

$$\Phi[y] = M[y] + k \ {}_a\!\int^b F(x, \xi)y(\xi)d\xi$$

$$= \lambda\{N[y] + k \ {}_a\!\int^b G(x, \xi)y(\xi)d\xi\} = \lambda\Psi[y]$$

is investigated. M and N are linear homogeneous differential forms:

$$M[y] = {}_\rho\sum\nolimits_0^m \mu_\rho(x)y^{(\rho)}(x), \quad N[y] = {}_\rho\sum\nolimits_0^n \nu_\rho(x)y^{(\rho)}(x);$$

$m > n$, λ is the unknown eigenvalue. The m linear homogeneous boundary conditions, $U_\rho[y] = 0$ $(\rho = 1,2,\ldots,m)$, are linearly independent. For a certain class of operators Φ and Ψ the eigenvalues can be determined, and an orthogonal relation can be established for two eigenfunctions associated with two eigenvalues. In the case of degenerated kernels the conditions are especially favorable. A numerical example dealing with vertical vibration of a suspension bridge,

$$y^{\mathrm{IV}} - 20y'' + k \ {}_{-1}\!\int^1 y dx = \lambda(1+x^2)y; \ y(\pm 1) = y''(\pm 1) = 0$$

is discussed.

Manfred Schaefer, Germany

1225. I. Y. Akooshski, "Some new methods of calculating sums of products on a tabulator" (in Russian), *Bull. Acad. Sci. USSR Ser. tech. Sci.* (*Izv. Akad. Nauk SSSR Ser. tekh. Nauk*), Aug. 1948, no. 8, pp. 1193–1228.

The author describes and compares several closely allied methods of calculating a sum ${}_i\Sigma_1^k a_i b_i$ on a punch-card tabulator. The main feature is that of writing one set of numbers, say the b's, in the binary system and putting them on cards by means of "x-punching." A detailed discussion of the calculation of the sum ${}_i\Sigma_1^k a_i b_i c_i$ follows, which is applicable, however, only to $c_i \leq 7$, but which is said to be capable of extension to larger values of c_i by carrying out the process in several stages. Some mention is made of the sum $\Sigma_1^k a_i b_i c_i d_i$, where two sets of numbers are written to the base 2, and of the sum $\Sigma_1^k a_i b_i^2$ [see also *Notes Acad. Sci. USSR* (*Doklady Akad. Nauk SSSR*), 1948, vol. 59, pp. 1521–1524].

Courtesy of Mathematical Reviews D. H. Lehmer, USA

1226. R. C. T. Smith, "The approximate solution of equations in infinitely many unknowns," *Quart. J. Math.*, Mar. 1947, vol. 18, pp. 25–52.

This paper investigates how closely the eigenvalues, λ, for the truncated equations

$$x_\alpha - \lambda \sum_{\beta=1}^{m} k_{\alpha\beta}x_\beta = 0 \ (\alpha = 1, \ldots, m)$$

approximate the eigenvalues of the infinite system of equations for which the summation extends from 1 to ∞. The method used is to reduce the infinite system of equations to a finite system by solving for $\lambda_{m+1}, \lambda_{m+2}, \ldots$ in terms of $\lambda_1, \ldots, \lambda_m$ by iteration and then eliminating. Comparison of the exact and truncated equations leads to a complicated inequality which establishes upper and lower bounds for the error in the eigenvalues computed by the truncated equations.

The method is applied to a system of equations obtained by C. M. A. Leggett for the elastic stability of a long curved strip under shear. A lengthy analysis and laborious calculations are required to determine numerical values of the error bounds. It is doubtful whether the applied mathematician will find the method practical in many cases.

Dana Young, USA

1227. Joseph Morris, "The escalator method in engineering vibration problems," John Wiley & Sons, Inc., New York, 1947. Cloth, 5.5 × 8.6 in., 270 pp., 64 figs., $4.50.

The foreword which G. Temple has written to this book leaves no doubt that the work done by the author in the field of structural and vibrational engineering is of great importance. It gives evidence of great appreciation for his original contributions both to the construction and development of powerful new methods, as well as to their practical application to numerous types of problem in aircraft engineering. Three researches are mentioned specifically: (1) the extension of the Hardy Cross method of moment distribution; (2) the development of vibration methods for the solution of statical or vibrational problems in systems with many members or degrees of freedom, and (3) the so-called escalator method, after which the book, under consideration here, has got its name. This third topic finds its treatment in chapters 10 and 11 of the book which therefore may be looked upon as its backbone. The essential characteristic feature of the method consists in the possibility to write the (generalized) secular equation $S_n = 0$ of the secular equation $S_{n-1} = 0$ of the $(n-1)$th order (which arises when the last row and last column of the secular determinant S_n are suppressed) and their corresponding characteristic functions. Therefore, if all characteristic quantities of $S_{n-1} = 0$ are known, the equation $S_n = 0$ at once can be written down explicitly as an algebraic equation of the nth order and consequently solved by using standard methods. Evidently it is required, for the solution of $S_n = 0$, to solve successively $S_1 = 0$, $S_2 = 0$, $S_{n-1} = 0$.

A reverse process is described, with the aim of writing the equation $S_{n-1} = 0$ in terms of the characteristic values and corresponding characteristic functions of the equation $S_n = 0$. The reader who may have special interest in the Hardy Cross method and connected iteration methods may be referred to chapter 6.

The book as a whole contains 22 chapters, the mere mention of which will convince the reader of the richness of its contents:

(1) Bending of rods. (2) The stressing of an aircraft undercarriage frame. (3) Slope deflection formulas. (4) Castigliano's theorem of least work. (5) The stressing of rigid jointed frames. (6) Hardy Cross moment-distribution methods. (7) Rayleigh's reciprocal theorems. (8) Methods of finding approximate frequencies of vibration. (9) The vibration of rotating shafts.

(10) The escalator process for the solution of Lagrangian frequency equations. (11) The escalator process for the linking up of frequency equations by parts. (12) The torsional vibrations of an engine crankshaft system. (13) The vibration of tapered rods resembling propeller blades. (14) Coupled engine torsional and propeller flexural vibrations. (15) The linkage by the escalator process of multiple propeller and gear systems. (16) Aircraft wing vibration. (17) Axial or longitudinal vibration of an aircraft. (18) The pitching and yawing flexural vibrations of an aircraft. (19) Forced vibration torques due to "firing" in an internal-combustion engine. (20) Periodicity of forced vibrations due to propeller blades passing "aerodynamic" obstacles. (21) Coupled engine torsional and propeller flexural vibrations in which account is taken of forced vibration and damping. (22) Pendulum dynamic vibration damper.

The reviewer regrets very much to say that the accuracy of expression and explanation is open to severe criticism (see for instance the second paragraph of chapter 10, which in its present form is quite unreadable). Were not the book of great importance both with regard to its scientific contents and to its practical use, this critical remark could have been suppressed. But it should be made for the sake of the second edition for which we certainly will not have to wait very long.

C. B. Biezeno, Holland

1228. Antonin Svoboda, "Computing mechanisms and linkages," *M.I.T. Rad. Lab. Ser.*, McGraw-Hill Book Co., New York, 1948. Cloth, 9.25×6 in., 359 pp. 177 figs.

This book is devoted to computers made entirely of mechanical elements. Such computers have been used for the rapid solution of gunfire control problems, in navigation and sometimes in other engineering applications. They are still the most economical, reliable and sturdy machines in those fields in which an accuracy of one part in a thousand is sufficient. A historical introduction describes the adding, multiplying, integrating, resolving and transforming components which have been customarily used in mechanical computers employing gears, cams, friction drives, worm drives, and linkage mechanisms. But the principal concern of this book is a detailed study of the simplest of the mechanical methods, namely, the linkage mechanisms made entirely of rigid bars joined by pivots.

The design of linkage mechanisms has been an empirical art in which the principal techniques were graphical. The largest part of this book is devoted to an exposition of these graphical techniques of which very few have been published previously. Good design of linkage computers requires a proper compromise between the complexity of the mechanism to make it follow the variations of an analytical or an empirical function and the need for simplicity to reduce the accumulation of errors due to play in the hinges. The mathematical part of the problem is usually the determination of the parameters to allow the solution to pass through a sufficiently scattered set of arbitrary points on the desired curve with the hope that the derived curve does not depart too much at other points.

A harmonic transformer consists of a crank driving a sliding rod. By varying the lengths of the crank and rod, the pivot points and the initial points of the motions, one obtains an enormous selection of available transformations. Convenient tables are given in a 50-page appendix for the selection of the desired function. Combinations of two or more harmonic transformers give closer approximations. A study of the effect of structural errors on the final accuracy of the mechanism is included.

The three-bar linkage (more frequently called the four-bar linkage) is then investigated. This permits more types of curves and functions since there are more parameters. Numerical tables and graphical diagrams of families of curves and nomograms are given to enable first-order design. Combinations of two and three harmonic transformers and a three-bar linkage are considered in several numerical examples.

Functions of two independent variables are mechanized approximately by means of linkages having two degrees of freedom. These are simpler and less expensive than the three-dimensional cams frequently used for this purpose.

Since the book is essentially a manual for designers, very few references are made to the literature. Important early work in the mathematical theory of approximating linkages was done by Burmester and Chebyshev. There has been much recent work by Z. S. Bloh and N. G. Bruyevich.

M. Goldberg, USA

1229. H. E. Rose, "The mechanical differential analyser: its principles, development, and applications," *Proc. Instn. mech. Engrs.*, 1948, vol. 159, no. 38, pp. 46–54, plus 2 plates.

This paper discusses the elements which are used in mechanical differential analyzers in general. Mechanisms such as integrator units, torque amplifiers, and frontlash units (to counteract backlash) are explained in detail. The Bush and Caldwell machine and the "Meccano" machine are discussed and shown in photographic views.

Henry J. Barten, USA

1230. J. G. LaBerge, "Some dimensional considerations in fluid mechanics," *Nat. Res. Counc. Canada Aero. Mech. Engng. Rep.*, no. MT-5, May 20, 1948, 80 pp.

This is a fairly extensive review of dimensional analysis and similarity, with particular emphasis on fluid-mechanical applications. It includes a broad range of specific examples, ending with the problem of data presentation for jet engines.

The examples are interesting but, unfortunately, the report seems to contain a few basic confusions. For instance, after the usual formal demonstration that the specific group of dimensionless parameters obtained depends upon the choice of "primary" physical quantities, the author states that such derived parameters as a^2/lg, $F/g\rho l^3$ and $F^2/\mu^2 gl^3$ are "meaningless" (a is the speed of sound). In fact, they are expressible as fundamental force combinations just as surely as are those parameters which have turned out to be of more practical importance (e.g., Reynolds, Froude and Mach numbers).

Another basic confusion is shown in the list of the fundamental types of forces that are met with in fluid-mechanics problems: along with the true fundamental forces, i.e., inertia, viscous, gravitational and elastic, the author lists "oscillating force," confusing the concept of time behavior of a force with the concept of its fundamental physical nature. The dimensionless "force ratio" he constructs using inertia force and "oscillating force" turns out to be simply a nondimensional kinematic variable, since he implicitly takes the oscillating force to be an oscillating inertia force.

Stanley Corrsin, USA

1231. Theodor Ricken, "Engineering handbook" (in German), Carl Hanser Verlag, Munich, 1949. Paper, 5.25×7.75 in., xiii + 292 pp.

This collection of tables and German (DIN) standards contains: the basic numerical tables; tables of units and physical constants; formulas of strength of materials; properties of metals and alloys; moments of inertia etc. for various beams; weight of sheet metal, wire, pipes, etc.; standards for bolts, screws shafts, wedges, gears, etc.; machining speeds and angles, drawing standards, tolerances, transmission devices, thermodynamic and heat transfer constants, basic electrical engineering formulas and standards, civil engineering standards and codes.

Ed.

⊛1232. Edward S. Allen, "Six-place tables," McGraw-Hill Book Co., New York & London, 1947. Cloth, 7.2 × 4.3 in., 232 pp., $2.50.

This is the seventh edition of a collection of tables of squares, cubes, square roots, cubic roots, etc., for integers from 1 to 1000, and of common and natural logarithms, trigonometric, exponential and hyperbolic functions, and logarithms of trigonometric functions to six decimals. To these are added short tables of elliptic and probability integrals and a small collection of integral formulas. Ed.

Mechanics (Dynamics, Statics, Kinematics)

(See also Revs. 1228, 1241, 1308, 1310)

⊛1233. Harvey Girvin, "A historical appraisal of mechanics," International Textbook Co., Scranton, 1948. Cloth, 6 × 9.25 in., 275 pp., 12 figs., $3.25.

"It is to be regretted," writes Dean A. A. Potter in a foreword to this volume, "that so little attention has been given in the literature of engineering to the historical aspects of mechanics." The present volume aims at giving engineers and engineering students a picture of the development of the science of Mechanics (part II), and of Mechanics of Materials (part III). It should be appraised as collateral reading for engineering courses, not as a textbook in history of science. Its amateurish quality is at once apparent from its style: the author calls Whewell alternately "Professor Whewell" and "Dr. Whewell," but on the other hand refers to Albertus Magnus as "Magnus"! References in French or German are frequently misspelled, sometimes with irresistibly funny results.

Factual misinformation is more serious. For example, in chapter I the author states that "the earth has been a dwelling place for some sort of human creatures for at least 25,000 years"! (no misprint here; on the same page 4 there are two more statements on the same chronological scale). The whole book is obviously a well-meaning, second-hand compilation. This may be unfortunate from the scholarly point of view, but does not necessarily detract from the usefulness of the work. There is a crying social need for humanizing the engineer, broadening his intellectual horizon and awakening his interest in the divergent directions of pure science and of history. This is what the present book aims at doing, and its unacademic, unassuming character may well be a virtue rather than a blemish. We hope that many an engineering student may find it in his departmental library, and opening its pages by chance, have his interest awakened in Leonardo or in Thomas Young, his ambition kindled by the names of contemporaries quoted in the last chapters. Therein lies the human value of Girvin's book. P. Le Corbeiller, USA

1234. H. Poritsky and D. W. Dudley, "Conjugate action of involute helical gears with parallel or inclined axes," *Quart. appl. Math.*, Oct. 1948, vol. 6, pp. 193–214.

Helical gears are used in modern applications to give smoother action by reducing the cogging action of the more familiar spur gears. The working surface of an involute helical gear is generated by a plane involute which moves with a uniform screw motion along the axis normal to the base circle at its center. The equations of this surface and some of its geometry are derived. It is shown to be the developable surface which is the tangent surface of a helix. The spherical indicatrix of the straight lines on this surface is a small circle. Two gears can operate together if the spherical indicatrix of one gear is tangent to, or intersects, the spherical indicatrix of the other. Several theorems are derived giving limitations on the parameters of mating pairs, the types of contact and the motions of the points of contact. Practical applications include the effects of the number of teeth, of displacements between axes and of improper angular relations. M. Goldberg, USA

1235. Theophile de Donder and Paul Melchior, "Gauss' principle of least action applied to the dynamics of rigid bodies with nonholonomic constraints" (in French), *C. R. Acad. Sci. Paris*, Nov. 15, 1948, vol. 227, pp. 1017–1018.

The equations of motion of a conservative nonholonomic system are obtained from the principle of least constraint (due to Gauss) with the apparent purpose of showing a connection with a modified least action principle.

Courtesy of Mathematical Reviews D. C. Lewis, USA

1236. Th. de Donder, "A new variational principle of dynamics of rigid bodies in nonholonomic rolling constraints" (in French), *Bull. Acad. Belg. Cl. Sci.*, 1948, vol. 34, no. 10, pp. 701–702.

The author sketches briefly the application of his new variational principle to the problem of a rolling hoop, a case in which the contraints are nonholonomic. The details of the method are given in a further paper by Paul Melchior. P. C. Dunne, England

1237. Paul Melchior, "On the dynamics of rigid bodies. II" (in French), *Bull. Acad. Belg. Cl. Sci.*, 1948, vol. 34, no. 10, pp. 779–784.

The author develops the equations of motion of a rigid body by means of the new variational principle of Th. de Donder. The paper continues the theory given in a paper previously reviewed [see Rev. **288**, March 1949, P. Melchoir, "Sur la dynamique des solides," *Bull. Acad. Belg. Cl. Sci.* 1948, vol. 34, No. 5, pp. 445–448]. P. C. Dunne, England

1238. Paul Melchior, "Dynamics of a rigid body in nonholonomic constraints according to the variational method of Th. de Donder" (in French), *Bull. Acad. Belg. Cl. Sci.*, 1948, vol. 34, no. 10, pp. 785–788.

The author applies the variational method of Th. de Donder to the problem of a hoop rolling on a plane. He shows that in the case of a circular hoop the method gives equations identical to those obtained previously by Paul Appel [*Traité de Mécanique Rationelle*, 1904, vol. II, pp. 243–244]. P. C. Dunne, England

1239. J. W. Klaren, "Some remarks on the technical requirements of modern railroad transportation and new methods for the calculation of the required power" (in Dutch with English summaries), *Ingenieur 's-Gravenhage*, 1949, vol. 61: Jan. 28, pp. V1-V8; Feb. 4, pp. V11-V20; Feb. 11, pp. V23-V29.

The paper begins with a discussion of the problem variables: the most economic velocity, properties of the several types of locomotives, design of several kinds of carriages, streamlining, etc. Then follows a classification assigning the most suitable type of service to externally powered electric trainsets, electric and Diesel locomotives.

Following a discussion of the allowable drive-axle loading and tractive effort, the author considers the train resistance comprising (1) the running resistance due to friction, oscillations, deflection of rails and, at great length, (2) the air resistance in head and side winds. Equations for the combined resistance are given for single carriages, trains, and streamlined trainsets. The author speci-

fies that a modern train powered by electric motors must be capable to be accelerated to 80 per cent of maximum velocity on a horizontal track within 10 min. The performance during acceleration is computed from the assumption of an acceleration diminishing linearly with speed for electric traction with external power supply. For Diesel electric traction the acceleration is assumed to be constant up to a speed of 15 km per hr and thence decreasing parabolically to zero at full speed. Examples are given of performance calculation for three basic types of power. Special consideration is given to electric trainsets with all, $^2/_3$, and $^1/_3$ of the axles driven. On steep grades the allowable maximum velocity, according to the author, may be taken at 60 per cent of maximum velocity on horizontal track and grades of 69 per cent when all axles are driven, or 22 per cent when $^1/_3$ of the axles are driven. This may be the maximum attainable for traction by adhesion between rail and wheel. F. Hymans, USA

Gyroscopics, Governors, Servos

1240. Winfried Oppelt, "Principles of automatic regulation" (in German), Wolfenbutteler Verlagsanstalt, Hannover, 1947. Paper, 8.25 × 5.75 in., 118 pp., 32 figs.

The book gives a very condensed and well-arranged discussion of general governing of linear systems. The use of many tabulated diagrams and block diagrams makes the book very handy. Three methods of study are presented: the method of differential equations, of transfer function, and of the response function to a unit impulsive input.

These three methods are applied to components as well as to their combinations. The over-all system is treated in the same way, and the stability criteria are derived using these three methods which lead to the criterion of Hurwitz, Cremer-Leonhard and Nyquist, respectively.

Some minor obscurities might be mentioned: for example, in the first line of table 13, or in the third line of table 14, where the scheme seems to show a centrifugal governor without any force of inertia.

The book is intended for undergraduate study. It is perhaps too concise for the purpose, but an extensive bibliography is provided. Mir. Nechleba, Czechoslovakia

Vibrations, Balancing

(See also Revs. 1227, 1331)

1241. A. A. Andronow and C. E. Chaikin, "Theory of oscillations," Princeton Univ. Press, Princeton, 1949. Cloth, 9.2 × 6 in., 358 pp., 313 figs., $6.

This book is a condensed translation (edited by Solomon Lefschetz) of a Russian book published in 1937. All of its contents were previously utilized in N. Minorsky, *Nonlinear Mechanics*, J. W. Edwards, Ann Arbor, Mich., 1947. This translation is more explicit in presentation and makes fewer demands on the reader. The work provides an introduction to the qualitative study of the vibrations of nonlinear systems of one degree of freedom. Phase-space representation is used throughout. Much attention is paid to self-excited vibrations and some of their most important applications.

The first four chapters deal with the most elementary theory of vibrations: linear systems, nonlinear conservative systems, and nonconservative systems. The fifth chapter is devoted to the presentation of the basic results of Poincaré and Liapounoff. The concepts of singular points and separatrixes are presented in detail. The Liapounoff stability concept is thoroughly discussed. These results are applied in various ways in the following chapter. The seventh chapter deals with discontinuous oscillations, parasitic parameters and stability. Certain related questions of degeneracy caused by stray impedance are discussed. The eighth chapter is devoted to systems with cylindrical phase surfaces. The ninth chapter deals with the quantitative investigation of nonlinear systems; van der Pol's method of approximation and some of its applications, and Poincaré's method of perturbations and some of its applications to vacuum tubes are discussed.

Of the three appendixes at the end of the volume the first is concerned with structural stability (i.e., retainment of the stability type when a parameter varies), a justification of van der Pol's approximation, while the third deals with van der Pol's equation for arbitrary values of the parameter. At the end of the book there is a reference list of the numerous practical applications treated in the text. These practical examples constitute one of the major attractions of this work. They deal mainly with vacuum-tube circuits, for the very practical reason that the range of vibration phenomena in mechanical systems is seriously limited. Enrico Volterra, USA

1242. M. Y. Leonov, "Certain criteria of dynamical stability" (in Russian), *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*), Nov.-Dec. 1948, vol. 12, pp. 737–748.

The author considers the equation

$$G(t)\, d^2z/dt^2 + (dG/dt + 2v(t))\, dz/dt + D(t)z = 0,$$

where all functions entering are real. A standard change of independent and dependent variable yields $d^2y/ds^2 + \theta(s)y = 0$. Solutions of the form $y = f(s) \sin \phi(s)$, $dy/ds = u(s) f(s) \cos \phi(s)$ are investigated, and relations are obtained between u, f and ϕ. After these preliminaries, the case where $\theta(s)$ is periodic with period a is treated. If for some n, $\phi(a) = \phi(0) + n\pi$, y is said to be a solution with "resonant phase." The conditions for the existence of such a solution are given, together with the connection between this concept and the boundedness or unboundedness of the solution. R. Bellman, USA

1243. S. M. Rytov and M. E. Zhabotinsky, "Application of the small parameter method to systems close to those of Sturm-Liouville" (in Russian), *Bull. Acad. Sci. USSR Ser. Phys.* (*Izv. Akad. Nauk SSSR Ser. Fiz.*), Mar.-Apr. 1947, vol. 11, pp. 135–140.

By making use of the method of small parameters first applied by Vitt [1934, USSR] and then by Strelkov [1935, USSR], the present authors transform the problem of self-excited oscillations of a slightly nonlinear arbitrary distributed system with nonlinear boundary conditions into the known problem of Sturm-Liouville. Both the damping and the departure from linearity must be small. The solution is written in terms of a Fourier series of which each coefficient itself is a series, restricted to the second order, in terms of the small parameter which expresses the deviation of the system from linearity. The authors discuss briefly the approach in determining the stability of such a system under a small disturbance superimposed on the periodic solution obtained. Applications suggested by the authors refer to electronic circuits and wave guides. Walter W. Soroka

1244. N. N. Baootin, "On the motion of an ideal clock model with two degrees of freedom. A model of the Galileo-Huyghens clock" (in Russian), *Notes Acad. Sci. USSR* (*Doklady Akad. Nauk SSSR*), July 1, 1948, vol. 61, pp. 17–20.

Andronov and Neimark [*C. R. Acad. Sci. URSS*, 1946, vol. 50, pp. 17–20] proposed a clock model preserving the basic fea-

tures of a clock as a system with two degrees of freedom and at the same time greatly simplifying computations. However, they only applied their scheme to clocks of pre-Galilean type, that is, without pendulum or spring. The author applies the same method to clocks of modern type, that is, with a pendulum or a spring. A procedure for obtaining the periodic motion is indicated, and stability conditions of rather general, but intricate, nature are given in the paper.

Courtesy of Mathematical Reviews S. Lefschetz, USA

1245. J. M. Burgers, "Note on the damping of the rotational oscillation of a spherical mass of an elastic fluid in consequence of slipping along the boundary," *Proc. kon. Ned. Akad. Wet.*, Feb. 1949, vol. 52, pp. 113–119.

This is a continuation of the study of damped oscillations of a spherical mass of elastic fluid [Burgers, *Proc. kon. Ned. Akad. Wet.*, 1948, vol. 51, p. 1211]. For systems having a smaller oleate concentration than those considered in the previous paper, the results do not agree with experiment and the author derives a more exact expression for the period and the logarithmic decrement, but satisfactory agreement still is not reached. The conclusion is that the damping is partly due to causes other than slipping along the walls of the vessel or the observed oscillations differ from the "normal oscillations" considered. Edward Saibel, USA

1246. G. de Vries and J. Rost, "Technical description of the Pseudyn balancing machine" (in French), *Off. nat. Étud. Rech. aéro. Rep.*, no. 21, 1949, 22 pp.

The early portions of this report are devoted to a general discussion of the problem of balancing aircraft-propeller installations. The various causes of static, dynamic and aerodynamic unbalance in aircraft propellers are described, and the nature of the vibrations induced in the aircraft while in flight and as a result of these unbalances is discussed briefly.

It is pointed out that, regardless of other precautions, balancing of a complete installation must generally be undertaken under flight conditions if satisfactory operation is to be achieved. While complete balance of an installation cannot be accomplished, it is almost always possible to determine, from flight measurements, a condition of static unbalance of the propeller which results in acceptably "quiet" operation over the range of normal flight conditions. The large volume of flight-test data required for this determination makes it essential that certain special test equipment be provided, to enable the balancing tests to be performed simply and economically.

Toward this end, a mechanical device called the "Pseudyn balancer" has been developed from an original Dornier idea, which permits all of the necessary data for a balancing study to be taken during a single flight test. The device is attached to the propeller hub and supplies a static unbalance, of controllable magnitude, which slowly rotates with respect to the propeller. The magnitude of the unbalance is at the control of the pilot, while the slow rotation of the unbalance relative to the propeller permits a study of the effects of phase. The mechanical construction of the Pseudyn balancer is described in detail in the paper.

The flight-test technique is to measure the aircraft vibrations induced by unbalances of various magnitudes and phases, as introduced by the Pseudyn device, and to search for the condition of most quiet operation. The proper amount of unbalance, both with regard to magnitude and phase, is then applied permanently to the propeller. Several illustrative vector-response diagrams, obtained during flight test, are given in the paper and show the effectiveness of the procedure. The interpretation of the flight-test data is discussed along with other practical aspects of the method.

It is urged that aircraft-vibration specialists study this report carefully, since it apparently offers a practical approach toward the alleviation of one of the more troublesome problems of the art.

Martin Goland, USA

Wave Motion, Impact

(See Revs. 1222, 1286, 1310, 1337, 1341)

Elasticity Theory

(See also Revs. 1255, 1256, 1263, 1266, 1271)

1247. A. N. Gleyzal, "A mathematical formulation of the general continuous deformation problem," *Quart. appl. Math.*, Jan. 1949, vol. 6, pp. 429–437.

The paper derives the relation essentially due to Killing (1870),

$$\eta_{ij} = \tfrac{1}{2}\dot{g}_{ij} = \tfrac{1}{2}(v_{i,j} + v_{j,i}),$$

where η_{ij} are the true-strain-rate components, $g_{ij}dx^i dx^j$ is the square of the line element, x^i the particle (Lagrange) coordinates, v^i the particle velocity vector and $v_{i,j}$ its covariant space derivative, relative to the metric g_{ij}. The dot indicates the particle derivative (at constant x^i).

The next concern of the author is to introduce the concept of the finite strain tensor ϵ_{ij}. A natural choice would be to define it by the equation $\dot{\epsilon}_{ij} = \eta_{ij}$ which leads to $\epsilon_{ij} = \tfrac{1}{2}[(g_{ij}(x,t) - g_{ij}(0,t)]$. However, the author, it seems, is not satisfied with this choice, for reasons unstated.

He proposes another choice of ϵ_{ij}, and a form of stress-strain law (involving integrals of g_i) which in one finite linear case ($x' = ax$, $y' = by$, $z' = bz$) gives the classical results. This the author regards as a sufficient justification of his choice. The reviewer feels that many other similar laws and definitions of ϵ_{ij} can be proposed that would lead to the classical results for all finite linear deformations. A. W. Wundheiler, USA

1248. Sigeiti Moriguti, "Fundamental theory of dislocation in an elastic body" (in Japanese), *Appl. Math. Mech.* (*Ōyō Sūgaku Rikigaku*), June 1947, vol. 1, pp. 29–36.

An elastic body which has initial stresses can be regarded as an elastic body with some dislocation or "incompatibility." The author introduces the "incompatibility" tensor R by the definition:

$$R = \begin{pmatrix} R_x U_z U_y \\ U_z R_y U_x \\ U_y U_x R_z \end{pmatrix}$$

where $$R_x = \frac{\partial^2\epsilon_z}{\partial y^2} + \frac{\partial^2\epsilon_y}{\partial z^2} - \frac{\partial^2\gamma_x}{\partial y\partial z}, \text{ etc.},$$

$$U_x = \frac{1}{2}\frac{\partial}{\partial x}\left(-\frac{\partial\gamma_x}{\partial x} + \frac{\partial\gamma_y}{\partial y} + \frac{\partial\gamma_z}{\partial z}\right) - \frac{\partial^2\epsilon_x}{\partial y\partial z}, \text{ etc.}$$

It is pointed out that dislocation and incompatibility in elasticity theory correspond closely with circulation and vorticity respectively in hydrodynamics, and the general theory of dislocation is developed on the basis of this remark.

As an example, a formula (an analog of the Biot-Savart formula) is derived which enables the initial stresses to be calculated when the distribution of the incompatibility R is given.

Takeo Mogami, Japan

Experimental Stress Analysis

(See also Rev. 1250)

1249. **N. J. Hoff, B. A. Boley, and J. M. Coan, "The development of a technique for testing stiff panels in edgewise compression,"** *Proc. Soc. exp. Stress Anal.*, 1948, vol. 5, no. 2, pp. 14–24.

A test jig and technique are described, used in determining the buckling loads of flat rectangular fiberglas panels subjected to edgewise compression. The strain distribution during the test was measured with resistance strain gages and controlled by means of a flexible loading strip supported on adjustable screws. Two sets of needle bearings were used to realize simple supports along the loaded edges. J. J. Koch, Holland

Rods, Beams, Shafts, Springs, Cables, etc.

(See also Revs. 1253, 1257, 1261, 1262, 1272)

1250. **J. B. Mantle and T. J. Dolan, "A photoelastic study of stresses in U-shaped members,"** *Proc. Soc. exp. Stress Anal.*, 1948, vol. 6, no. 1, pp. 66–73.

A great number of photoelastic tests were made on U-shaped beams with the loads applied perpendicular to the two arms. The results are given in tables and plotted in graphs, which will be greatly useful to designers. The material used was B.T. 61–893. J. J. Koch, Holland

1251. **Franco Levi, "Nonhomogeneous beams in viscous state"** (in Italian), *G. Gen. civ.*, Oct. 1948, vol. 86, pp. 556–567.

The author considers nonhomogeneous beams composed of two different materials; for instance a concrete beam combined with a tile (eventually prestressed) plate: shrinkage and viscous flow in the concrete part produce stresses on the whole cross section.

Shrinkage and specific viscous flow are assumed as exponential functions of time; also viscous flow is taken as a linear function of stress. It is assumed that originally plane cross sections of the beam remain plane, while stresses are statically equivalent to the bending moment M due to external forces ($M = 0$ if only shrinkage is considered). The strain diagram is thus defined through the unit contraction $\bar{\lambda}$ and the curvature $\mu = M/EJ + \bar{\mu}$. The author, extending Colonnetti's method, establishes and solves a system of two linear differential equations with constant coefficients; the functions $\bar{\lambda}(t)$, $\bar{\mu}(t)$ are obtained.

In a particular case deflection and stresses were evaluated, the viscoelastic assumption being confirmed by tests. However, stresses due to shrinkage were 40 per cent low owing to viscous flow.

Differential equations are also established for statically indeterminate beams, but in this case coefficients are no longer constant, because the bending moment and the tension in the cross section depend upon time; hence the integration offers great difficulties. D. Gentiloni-Silverj, Italy

1252. **E. P. Popov, "Bending of beams with creep,"** *J. appl. Phys.*, Mar. 1949, vol. 20, pp. 251–256.

The subject of the paper is the calculation of the creep deformation of a bent beam in the transient range, that is, before a steady-state creep is reached. The proposed method consists of a numerical step-by-step procedure which, at any instant, satisfies the equilibrium conditions and the constancy of the planeness of the cross sections in the beam under pure bending. It is shown that as creep advances, the magnitude of the initially elastic stresses in the extreme fibers is somewhat reduced, and more of the stress is shifted toward the interior. This transient period later merges into the steady-state creep, where the creep rates in the fibers are proportional to their respective distances from the neutral axis. M. Hetényi, USA

Plates, Disks, Shells, Membranes

(See also Revs. 1255, 1266, 1276)

1253. **Eric Reissner, "Small bending and stretching of sandwich-type shells,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1832, Mar. 1949, 89 pp.

The classical small-deflection theory of thin elastic shells is extended to shells of sandwich materials which consist of a core layer of thickness h with elastic constants E_c and G_c, and two face-layers of thickness t and elastic constants E_f, G_f and ν_f. It is assumed that the ratios t/h and $E_c h/E_f t$ are both small compared with unity, and therefore only the transverse normal and shear stresses in the core layer are retained in the analysis. The faces are treated as membranes and the core is taken as a three-dimensional body. The strain energy of the composite system is formulated and Castigliano's theorem of minimum complementary energy is used to arrive at the relations between the stress resultants and the deformation of the shell.

Specific applications are made to (1) flat plates, (2) circular rings, (3) circular cylindrical shells, and (4) spherical shells with axisymmetric deformation. Numerous examples are given to illustrate the probable magnitude of the effects of the transverse shear stress and the transverse normal stress.

It may be noted that the significant parameter involved is either $[(h + t)t/a^2][E_f/G_c]$ or $[(h + t)t/a^2][E_f/E_c]$, a being a typical dimension of the shell, such as the width or the radius of the plate or the mean radius of the shell. Transverse normal stress needs to be considered only for the case of shells and circular rings. Conrad C. Wan, USA

Buckling Problems

(See also Revs. 1226, 1249, 1276)

1254. **Norman Grossman, "Elastic stability of simply supported flat rectangular plates under critical combinations of transverse compression and longitudinal bending,"** *J. aero. Sci.*, May 1949, vol. 16, pp. 272–276.

The title problem is solved theoretically and the results are given by means of curves of critical bending-stress coefficient vs. plate-aspect ratio for constant compressive stress ratio, and by interaction curves. From a practical viewpoint probably the fact of greatest interest is that the interaction curves change very markedly as the aspect ratio changes. From a theoretical viewpoint the feature of greatest interest is the setting up of the stability determinant.

It is found that the equations relating the Fourier deflection coefficients a_i and buckling parameter γ take the forms $\gamma a_i = {}_j\Sigma A_{ij}a_j$ and $\gamma a_j = {}_k\Sigma B_{jk}a_k$ where i and k are odd and j even, so that $\gamma^2 a_i = {}_{jk}\Sigma A_{ij}B_{jk}a_k$. The last equation involves only odd deflection coefficients, but the effects of the even coefficients have also been taken into account. The feasibility of this is typical of buckling problems involving shear or bending, the eigenvalues of which occur in pairs of numbers of opposite signs and the same absolute values.

The author uses the procedure outlined to obtain results from a second-order matrix for the case in which the first four Fourier coefficients are nonvanishing. It may be of interest to note that

in problems of this type additional accuracy can be obtained with comparatively slight effort by considering, for instance, a few nonvanishing even coefficients and many odd ones. By eliminating the many odd coefficients in the manner indicated above, the resulting matrix has an order equal to the small number of even coefficients. S. B. Batdorf, USA

1255. D. M. A. Leggett and R. P. N. Jones, "The behavior of a cylindrical shell under axial compression when the buckling load has been exceeded," *Rep. Memo. aero. Res. Counc. London*, no. 2190, Aug. 1942 (issued in 1948), 11 pp.

Von Kármán and Tsien had indicated in 1941 that a thin cylindrical shell can be maintained in a buckled state by a compressive load considerably smaller than that previously predicted by Southwell. The present paper is an extension of the work of von Kármán and Tsien and its object is to derive quantitative results. While in the former investigation, out of the four parameters which define the slope and the amplitude of the buckles, two were assumed arbitrarily, in this paper all four parameters are determined from minimum-strain-energy conditions. It is assumed throughout that the stresses in the buckled shell remain within the elastic range of the material. The principal conclusions of the paper are that an axially loaded cylindrical shell can be maintained in a buckled condition by a load which is about one-third of the critical compressive load obtained by Southwell, and that the buckled cylinder has only about one-quarter of its original stiffness. Numerical values of the buckling load are given for cylinders of various proportions, and it is shown that the slope of the diamond-shaped buckles changes and they become wider as the strain is increased, while there is a small variation in the number of circumferential waves. M. Hetényi, USA

1256. Sigeiti Moriguti, "Apparent modulus of elasticity of a circular plate just after buckling" (in Japanese), *J. Soc. appl. Mech. Japan*, May 1948, vol. 1, pp. 81–86.

A uniformly compressed circular plate is clamped (case A) or simply supported (case B) along the edge. Assuming the shape of the buckled plate to be rotationally symmetric, the author calculates the apparent modulus of elasticity E' just after buckling, by the method of perturbation. The "incompatibility" introduced by the author [*Appl. Math. Mech.* (*Ōyō Sūgaku Rikigaku*, June 1947, vol. 1, pp. 29–36; see Rev. **1248**, in this issue] is assumed to be either zero (corresponding to no initial stresses) or constant (corresponding to the initial stress distribution $\sigma_r = \lambda(r^2 - a^2)$, a being the radius of the plate). In the absence of incompatibility, $E' = 0.387\,E$ (case A) or $E' = 0.173\,E$ (case B). In the presence of a distribution of incompatibility lowering the buckling load, E' is slightly greater than the above values. Isao Imai, Japan

1257. G. C. Best, "The theory of beam columns with continuous support applied to the problem of stringer roll," *J. aero. Sci.*, May 1949, vol. 16, pp. 289–295.

Beam-column theory is extended to include the effect of continuous elastic support. The mathematical procedure used is clearly presented and the resulting equations are potentially of considerable interest and value to engineers confronted by beam-column problems.

Equations are given for the bending moment when the roots of the expressions required in its determination are complex, equal and imaginary, or unequal and imaginary; but the paper contains no statement as to the physical limitations of systems of loads and supports compatible with each set of roots. The practical significance of each of the three cases considered is left for the reader to interpret. Procedures are included for the determination of the points at which maximum moments occur and, in the appendix, for the computation of stiffness and carry-over factors, and of buckling loads.

A practical application of the method is given. In it, part of a J-stringer is assumed to behave as a beam column, while the remainder of the stringer and the sheet to which it is attached are assumed to contribute elastic support. The method of dividing the stringer between column-area and support-area is vague, the effect of nonsymmetry of the column-area is ignored, and some of the data introduced are so poorly defined that from this example it is difficult to determine the general suitability of the method for problems involving stringer roll. Those desiring to check this example should note that the centroid of the column-area is 0.5781 in. to the left of, and 0.225 in. below the upper right corner of that area, instead of 0.5775 in. and 0.2 in. as indicated in Fig. 2 of the paper. Joseph S. Newell, USA

Joints and Joining Methods

1258. W. Muckle, "The distribution of load in riveted joints," *Shipbuilder*, Apr. 1949, vol. 56, pp. 225–228.

A study is made to determine the load distribution in a riveted joint by use of the principle of minimum strain energy. The strain energy of a portion of the plating lying between the extreme rows of rivets in a joint is added to the strain energy due to shear of the rivets. The condition for minimum strain energy is applied to derive a sufficient number of additional equations to supplement the equations of equilibrium and to determine the load distribution. The analysis shows that the load is not uniformly distributed over the various rows of rivets, the outer rows taking more, and the inner rows less, than the average load. R. L. Bisplinghoff, USA

1259. E. G. West, "Welding of aluminum and its alloys," *Engineering*, 1949, vol. 167: Apr. 8, pp. 317–319; Apr. 15, pp. 341–342.

This is a critical review of current British practice and probable developments, covering gas, arc, argon-arc, pressure, resistance spot, resistance seam, and flash welding and brazing, including the process and the properties of the welds; it gives reference to recently published research and subjects of current research, particularly at Birmingham University. Pressure welding is highly recommended. Benjamin Miller, USA

Structures

(See also Revs. 1227, 1253, 1257, 1276, 1310, 1334)

⊛1260. St. Błaszkowiak, "Cross's method" (in Polish), Instytut Badawczy Budownictwa, Warsaw, 1948. Paper, 12.5 × 8.75 in., 130 pp., price unknown.

The procedure of structural analysis known in this country as the Hardy Cross method has only recently caught the attention of European engineers, particularly those who, during the war years, found refuge in the Anglo-Saxon countries. Although the difference between the Cross method and most of the methods of analysis used for many years under various names by engineers in Central and Western Europe is very slight, it is apparently considered sufficient to warrant the illustrative treatment it receives in the present monograph written with the direct purpose of instructing Polish engineers in its practical use. After a very con-

cise general outline of the method, 82 examples of structures, including various frames, frame girders, continuous arches, etc. under vertical and lateral loads, which have been selected from various, mostly European, textbooks of structural analysis, are treated in detail by the Cross method; its applicability to problems of buckling and mechanical vibrations is also illustrated by several examples. A. M. Freudenthal, USA

1261. **T. M. Yu, "Continuous arches and bents analyzed by column analogy,"** *Proc. Amer. Soc. civ. Engrs.*, Feb. 1949, vol. 75, pp. 167–184.

Hardy Cross's column analogy, applicable as originally presented to beams and structures having no more than three redundant elements, is in this paper extended to structures of a higher degree of redundancy. This is accomplished by cutting from the structure a single member, such as a beam, arch or bent, between nodal points, and considering the adjoining portions of the structure as supports for the removed member. Each of these portions is then replaced by an elastically equivalent support, represented by an extension of the beam, arch or bent. The method is applicable only in cases where, upon the removal of the beam, arch or bent, the adjoining portions of the structure are independent of each other.

In a numerical example the method is applied to a three-bay bent, with the center bent of greater height than the two equal outer bents. The author also employs the column analogy in the determination of influence lines for a three-bay continuous bent. F. Hymans, USA

1262. **Arthur G. Boorman, "An estimate of stresses produced in reinforced-concrete members under working loads,"** *J. Instn. civ. Engrs.*, Dec. 1948, vol. 31, pp. 101–116.

On the basis of data for the shrinkage and creep of concrete, presented by W. H. Glanville (*Studies in Reinforced Concrete*, London, 1930), the author examines the changes in the stress distribution which arise in the course of time in reinforced-concrete structures. In singly reinforced beams the state of stress is affected but little by the foregoing phenomena. On the other hand in the presence of reinforcement on both sides, the state of stress induced by bending, axial compression, and combined compression and bending in time approaches that which would occur if the action of the concrete were neglected. Shrinkage and creep thus lead to stress relief of the concrete in the compression zone. F. Stüssi, Switzerland

Rheology (Plastic, Viscoplastic Flow)

(See also Revs. 1247, 1251, 1252, 1262, 1269, 1280, 1332, 1333)

1263. **Heinrich Brandenberger, "A new theory of elasticity and strength of materials"** (in German), Schweizer Druck-und-Verlagshaus, Zurich, 1948. Cloth, 8.2 × 5.7 in., 216 pp., 64 figs., $4.66.

The rather ambitious title of this book covers an attempt to explain the most important phenomena of deformation and strength of materials in terms of the classical elastic theory combined with the maximum-strain criterion of plastic flow, and the concept that such flow is not isotropic but takes place in the direction in which the strain has exceeded a critical value, while the material remains elastic in the directions in which this value has not been attained.

The author bases his theory on the assumption that splitting up of the general state of elastic strain into a volumetric and a distortional component is sufficient to bring classical elastic theory into perfect agreement with the observed behavior of real materials; he also considers that a Poisson ratio of 0.3 is characteristic for such materials under any condition of strain. On the basis of these assumptions the author attempts an explanation of such a complex phenomenon as the "Bauschinger effect" and is actually able to show that his theory reproduces the results of tests on brass under reversed loading "partly with an accuracy of zero per cent."

There is no mention in the book of any of the current theories of plasticity, work-hardening, fatigue, anelasticity, etc., nor is any original paper in any of those fields mentioned in the reference list. This is hardly surprising since the author seems fully convinced (p. 10) that he has succeeded in providing the correct answer to any strength problem which can arise, and that from now on nothing more has to be done but to apply the formulas conveniently listed in his book. A. M. Freudenthal, USA

1264. **S. B. Batdorf and Bernard Budiansky, "A mathematical theory of plasticity based on the concept of slip,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1871, Apr. 1949, 33 pp.

Recent studies of the mathematical theory of plasticity have been concerned with a generalization of the mathematical relation between stress pattern, strain and strain rate. Many workers in allied fields have been skeptical as to the ultimate success of such mathematical generalizations in deducing relations valid for other than a limited range of conditions.

To these workers the present research is a refreshing change. This work abandons all attempts at finding concise mathematical relations, but rather starts from the empirical fact that all deformation occurs by slip across certain crystallographic planes. The authors then assume certain simple laws of work hardening, and compute the consequences of such assumptions. In the cases they have considered, these computations lead to better agreement with observed results than the past theories based upon formal mathematical relations.

While it may no doubt turn out that the assumptions of the authors regarding the elementary process of strain hardening may have to be modified, there can be little doubt that their general approach will turn out to be exceedingly fruitful. Clarence Zener, USA

1265. **H. J. Greenberg, "Complementary minimum principles for an elastic-plastic material,"** *Quart. appl. Math.*, Apr. 1949, vol. 7, pp. 85–95.

This paper is concerned with minimum principles for an incompressible material satisfying the Prandtl-Reuss rate theory of plasticity in the range of infinitesimal strain. The author proves first that, compared with all suitably defined "admissible" stress-rate deviators, the stress rate of this theory yields a minimum for the volume integral of its intensity minus a certain surface integral. Second, the author establishes a similar minimum principle for the strain rate. These two principles are complementary in that for an actual solution of the equations the two quantities minimized become equal. C. A. Truesdell, USA

1266. **L. A. Galin, "An analogy for a plane elastoplastic problem"** (in Russian), *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*), Nov. 1948, vol. 12, pp. 757–760.

An infinite plate with a hole of arbitrary shape L is in a state of plane strain, and is subjected to uniform loading at infinity characterized by the principal stresses $\sigma_{x\infty} = A$, $\sigma_{y\infty} = B$. The boundary L is free from tractions. The stresses at an arbitrary point of the plate have the values

$$\sigma_x = \partial^2\varphi/\partial y^2,\ \sigma_y = \partial^2\varphi/\partial x^2,\ \tau = -\partial^2\varphi/\partial x\partial y,$$

where φ is the stress function.

When the plate is elastic, the differential equation $\Delta\Delta\,\varphi = 0$ holds with the conditions $\partial^2\varphi/\partial s^2 = 0$, $\partial^2\varphi/\partial s\partial n = 0$ on the boundary L. For $r = \sqrt{x^2 + y^2} \to \infty$, $\varphi \to Bx^2/2 + Ax^2/2$. These same equations substituting the displacement w for φ apply also to the bending of an elastic plate which is rigidly clamped along its boundary L and is loaded by the proper moments sufficiently far from the hole. Thus the analogous deflection function w coincides with the Airy function in two-dimensional elasticity.

If we now consider the elastoplastic condition of the same plate, we must first determine the stress function φ_1 in the plastic region from the plasticity condition

$$\left(\frac{\partial^2\varphi_1}{\partial x^2} - \frac{\partial^2\varphi_1}{\partial y^2}\right)^2 + 4\left(\frac{\partial^2\varphi_1}{\partial x\partial y}\right)^2 = 4k^2$$

with boundary conditions

$$\partial^2\varphi_1/\partial s^2 = \partial^2\varphi_1/\partial s\partial n = 0,$$

where k is the plasticity constant. In the case of a circular hole of radius R we have $\varphi_1 = k[r^2\ln(r/R) - (r^2/2) + (R^2/2)]$. In his book *Plasticity Theory* (in Russian, 1946), W. W. Sokolovsky gives the functions φ_1 for several holes of polygonal shape.

In order to make use of the elastoplastic analogy experimentally, we must construct two rigid surfaces φ_1 and $-\varphi_1$ with the same boundary curves L, and place the elastic plate between them. If we now load the plate with moments of suitable magnitude and sufficiently far from the curve L, the plate is divided into two regions. In one, it touches the surface φ_1 or $-\varphi_1$, that is the region corresponding to the plastic zone of the plate. The remainder corresponds to the elastic zone of the plate. The reviewer feels that this analogy has the disadvantage, from an experimental point of view, that the function φ_1 cannot be obtained purely by experiment. W. Burzynski, Poland

1267. I. A. Oding, "On the role of dislocations in the creep process" (in Russian), *Bull. Acad. Sci. USSR. Ser. tech. Sci.* (*Izv. Akad. Nauk SSSR Ser. tekh. Nauk*), Dec. 1948, no. 12, pp. 1795–1802.

The author assumes that creep is the result of simultaneous hardening and recovery of a metal. Since plastic deformation consists of the movement of dislocations, it is at first facilitated by an increasing number of such dislocations. If the number of dislocations increases further, their interaction causes strengthening of the metal. The strength therefore has a minimum value for a certain intermediate number of dislocations.

The formation of dislocations is increased by microcracks and by thermal fluctuations. Their number is decreased by mutual neutralization of dislocations of opposite sign and by their dissolution at the mosaic block surface.

A metal with a small number of dislocations, in creeping at high temperatures and under low stresses, will be weakened by the plastic deformation and hardened by the removal of dislocations, the resulting behavior depending on the interaction of these two processes. At low temperatures and high stresses the formation of dislocations will accelerate the creep, in accordance with experimental results. A metal with a high density of dislocations will be strengthened by creep, its creep rate being lowered.

This explains the well-known fact that the creep rate of a work-hardened metal is lower at low temperatures and higher at high temperatures than the creep rate of an annealed metal (steel at 500 C and 600 C). The author considers that the common assumption that the difference is due to recrystallization is wrong.

George Masing, Germany

1268. F. A. Bakhshiyan, "Rotation of a rigid cylinder in a viscoplastic medium" (in Russian), *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*), Nov.-Dec., 1948, vol. 12, pp. 749–756.

The author considers viscoplastic flows past a solid rotating cylinder. He obtains the equation

$$u_\tau = \nu_1(u_{xx} + u_x/x - u/x^2) - 2/(Rx).$$

Here τ, u, x are nondimensional time, velocity and distance from the cylinder axis, respectively, R the Reynolds number and $\nu_1 = ka^2\rho/\mu^2$, where k is the plasticity constant, a the cylinder radius, ρ density, and μ the viscosity. The boundary conditons are stated.

For the case of a bounded medium and uniform rotation a solution of the form $u(x,\tau) = u_1(x) - X(x)\,T(\tau)$ is found, with

$$u_1 = c_1x + c_2/x + S(\log x - 1/2),$$

V being the velocity $u = 1$, and $S = ak/\mu V$.

$$X = J_1(\alpha x) + gN_1(\alpha x),\ T = A(\alpha)\exp(-\alpha^2\nu_1\tau),$$

where J_1 and N_1 are the Bessel functions of order one, and g, α, $A(\alpha)$ are constants. If b is the radius of the cylindrical plastic flow boundary, and $\lambda = b/a$, the boundary conditions require that, for any integral n,

$$\alpha \approx n\pi(\lambda - 1),\ A = \lambda\pi\alpha^{-2}N_1(\alpha)N(\lambda\alpha)[N_2^{-2}(\alpha) - N_1^2(\lambda\alpha)]^{-1}$$
$$[\lambda(2S - \alpha^2)N_1(\lambda\alpha) - 2SN_1(\alpha)],$$

where $N\pi^2\alpha^2/2 = N_1^{-2}(\lambda\alpha) - N_1^{-2}(\alpha)$ Considerable quadrature computation is involved, here and later.

For an arbitrary rotation in a bounded medium the author sets

$$u = -2\tau/Rx + \omega(x,\tau),\ \text{and, with}\ \Omega(x,p) = p\,{}_{-\infty}\!\int^{\infty} e^{-p\tau}\omega(x,\tau)d\tau,$$

obtains

$$\Omega_{xx} + \Omega_x/x - (\chi^2 + x^{-2})\Omega = 0$$

where $\chi^2 = RSp$. Ω is expressed in terms of Bessel functions of both kinds and the first order.

For an infinite medium and uniform rotation Ω is expressed in terms of Bessel functions of the second kind, and, for large χ, in terms of exponentials. For small τ, $\omega(x,\tau)$ is expressible in terms of the probability integral and its quadratures.

Other solutions in terms of combinations of Bessel with arbitrary functions, but not meeting the boundary conditions, are mentioned. They solve the problems of gradual and sudden motion from the state of rest, if the arbitrary functions satisfy appropriate Volterra-type integral equations.

George Masing, Germany

Failure, Mechanics of Solid State

(See also Revs. 1274, 1332, 1333, 1344)

1269. G. V. Oozhik, "On the unification of failure and yield conditions" (in Russian), *Notes Acad. Sci. USSR* (*Doklady Akad. Nauk SSSR*), Feb. 1, 1949, vol. 64, no. 4, pp. 471–474.

Theories of strength are based at present on either normal or shear stresses. Cohesive failure due to normal stresses is commonly observed in brittle materials, and plastic deformation due to shear stress in ductile materials, but the cohesive resistance R_0 in ductile materials elastically deformed up to failure has so far not been taken into account. The author has shown in his earlier

work [*Bull. Acad. Sci. USSR Ser. tech. Sci.*, 1948, no. 10, p. 1547; see Rev. **477**, Apr. 1949] that R_σ can be determined in ductile materials by an analysis of the fracture of a notched bar. There are parts with brittle fracture and others with plastic deformation, showing that the occurrence of one or the other depends on the stress conditions in a plastic body.

If the principal stresses are $\sigma_1 > \sigma_2 > \sigma_3$ the author's complete conditions for failure or yielding are: (1) plastic flow occurs when $\sigma_1 - \sigma_3 = \sigma_s$, where σ_s is the yield limit and $\sigma_1 < R_\sigma$; (2) brittle cohesive failure is obtained if $\sigma_1 = R_\sigma$ and $\sigma_1 - \sigma_3 \leqq \sigma_s$.

The conditions for cohesive failure in ductile materials after plastic deformation have so far not been analyzed, according to the author. George Masing, Germany

1270. **F. H. Keating, "Chemical manifestations of internal stress,"** *Inst. Metals Monogr. Rep. Ser.*, no. 5, 1948, pp. 311–331.

Stress-corrosion cracking is a combined effect of internal stresses and a corrosive environment, in which the effect of the stresses is presumably confined to the development of purely mechanical damage. According to the author, the localized corrosion attack results in regions of stress concentrations, which under tensile loading give rise to the initiation of cracks. When a crack is arrested at some obstacle (nonmetallic inclusion, grain boundary, etc.), it takes some time before fresh corrodent will reach the point of the crack and can start a new attack. This mechanism may account for the branching of cracks and the alternately transgranular and intergranular appearance of fractures, which have often been observed. J. A. Haringx, Holland

Design Factors, Meaning of Material Tests

(See Revs. 1271, 1272, 1273, 1274)

Material Test Techniques

(See also Revs. 1274, 1278, 1344)

1271. **Else Holm, Ragnar Holm, and Earle I. Shobert II, "Theory of hardness and measurements applicable to contact problems,"** *J. appl. Phys.*, Apr. 1949, vol. 20, pp. 319–327.

The paper reviews the theory of hardness as measured by the older accepted test methods, and brings out the difficulties present in the Rockwell, Brinell and Vickers tests. The ball-indentation test is recommended for contact problems, with hardness defined as the ratio of the contact load to the mouth area. The latter is presented as a function of the depth and radius of curvature of the indentation. The variation of hardness so defined with geometrical and metallurgical conditions is discussed, together with its relationship to the yield point of the material.

S. K. Ghaswala, India

1272. **E. Sperling, "Tests on endurance of railway car axles as a basis for their design"** (in German), *Z. Ver. dtsch. Ing.*, Mar. 15, 1949, vol. 91, pp. 134–136.

This is a report on axle endurance tests made by two methods. In the first method the axle was loaded in the same manner as in service, and was rotated; in the second, one wheel was clamped and a vibration generator was used to deflect the other end in a circular path. The material used was steel. Only the faired curves from the experimental results are given, with no scatter shown. The endurance limit of the unnotched material is assumed, from work by Lehr, to be 16 kg. per sq mm. The ratio to that value of the nominal stress at the endurance limit in the assembly of axle and wheel is taken as the notch factor.

Fifty assemblies were tested, with various combinations of fillets and diameters of the free shaft and of the seat on which the wheel is fitted. Since this seat must be larger in comparison to the shaft when larger fillets are used, it turns out that the notch factor rises with increasing fillet radius when referred to the diameter of the seat.

A fillet with radius increasing inboard is preferred. It is shown that under a load of 20 tons, diameters of 160 and 180 mm for the shaft and seat are adequate. The previously accepted value of 185 mm for the seat diameter thus has ample overload capacity.

W. P. Roop, USA

Mechanical Properties of Specific Materials

(See also Revs. 1259, 1262)

1273. **T. J. Dolan and C. S. Yen, "Some aspects of the effect of metallurgical structure on fatigue strength and notch-sensitivity of steel,"** *Proc. Amer. Soc. Test. Mat.*, 1948, vol. 48, pp. 664–695.

The main purpose of the present investigation was to determine the influence of different types of heat-treating upon the fatigue properties, notch-sensitivity and notch-impact strength of one plain carbon and two alloy steels (SAE 1045, 2340, and 3140) with the same hardness $R_B = 101$. The steels were heat-treated by two different methods. In order to obtain a type of structure with martensite or ferrite plus pearlite the specimens were rapidly or slowly cooled in quenching and then tempered. In an additional test series the specimens of an alloy steel (SAE 2340) were brought up to a hardness of $R_c = 27$ by quenching and tempering and by austempering.

The flexural fatigue tests were made on small unnotched polished and notched (60-deg-V-notch) specimens on two Krouse rotating cantilever beam machines (n = 6000–8000 rpm); the final number of cycles ranged in these tests from 10 to 70 millions. For comparison with the rapid drop of the Charpy test, specimens were tested in the range of temperature from −212 F to 220 F.

These test results should be generalized by more detailed test series, containing the metallurgical influences, several heat treatments, surface conditions, residual stresses, etc. The following conclusions can be drawn from the test results of the materials and heat treatments employed. The yield ratio (i.e., the ratio of yield strength to tensile strength) of the specimens having a structure of primarily tempered martensite is higher than for the specimens with a structure of ferrite plus pearlite. The endurance limits of the unnotched and notched specimens and the Charpy impact values are greater for the alloy steels rapidly cooled in quenching. The strength reduction factor (i.e., the ratio of the endurance limits of unnotched to notched specimens) for specimens of the tempered martensitic steels is low when compared with those for the same steels heat-treated to a structure of ferrite plus pearlite. Furthermore the test results show that, on account of the differences in the rate of strain, kind of loading, and fracture, there is no direct correlation between the notch-sensitivity in a fatigue and a Charpy impact test. M. Hempel, Germany

1274. **Karl Heinz Leise, "The connection between the hardness of a cold-worked surface and the structural change due to the cold-working"** (in German), *Metallforsch.*, Apr. 1947, vol. 2, pp. 111–114.

The author has determined the surface microhardness of single crystals of Cu, Al and Fe, in the unwork-hardened state (electrolytically polished) and after work hardening; in the latter case he determined the depth at which the hardness returns to its unwork-hardened value. In addition the author has studied, by

removing thin layers and by the use of electron-diffraction pictures, the depth at which the single crystal image is reobtained. A comparison of these results indicates that these two depths are nearly the same, which indicates that hardening produced by cold-working is due to grain fragmentation. The experimental technique described is interesting. Henri M. Schnadt, Luxemburg

1275. Howard C. Cross and J. W. Freeman, "Office of Naval Research and NACA metallurgical investigation of a large forged disc of Inconel X alloy," *Nat. adv. Comm. Aero. tech. Note*, no. 1770, Apr. 1949, 31 pp.

This report gives the results of an investigation of the properties of a large forged disk of Inconel X alloy at various elevated temperatures. The purpose of this study was to determine the properties required for rotor disks of gas turbines. The data listed include the results of tensile, impact, rupture, time-deformation, creep, and structural-stability tests. The Inconel X alloy shows superior properties at 1200 and 1350 F for time periods up to 1000 hours, compared to low-carbon alloys.

Joseph Marin, USA

1276. A. G. H. Dietz, "Engineering laminates," John Wiley and Sons, New York, 1949. Cloth, 9.25 × 6 in., 797 pp., 337 figs., $10.

Chapter 1 ("The strength of laminates and sandwich structural elements," by N. J. Hoff) presents a thorough account of present structural theory with a comprehensive survey of the literature up to 1946. The other chapters contain a great quantity of data on specific materials and their applications.

W. T. Koiter, Holland

1277. Walter H. Price and William A. Cordon, "Tests of lightweight-aggregate concrete designed for monolithic construction," *J. Amer. Concr. Inst.*, Apr. 1949, vol. 20, pp. 581–600.

This paper summarizes Bureau of Reclamation test results and recommendations concerning lightweight aggregates produced in the western United States. The characteristics and origin of lightweight aggregates are described and the comparative results of laboratory tests of strength, insulating value, shrinkage, and weathering resistance of concrete made with lightweight aggregates from 17 producers are summarized. Data are included on expanded shale, expanded slag, scoria, pumice, perlite, exfoliated vermiculite and diatomaceous earth.

The strength and thermal conductivity of lightweight-aggregate concrete are shown to be roughly proportional to its density; disruption from alkali-aggregate reaction is not discussed.

C. Martin Duke, USA

1278. I. Barducci and G. Pasqualini, "Measurement of the internal friction and the elastic constants of wood" (in Italian with English summary), *Nuovo Cim.*, Oct. 1948, vol. 5, pp. 416–446.

This paper discusses an apparatus for making tests on vibrating wooden rods, usually of the dimensions 10 × 1.0 × 0.2 cm. The vibrations are excited by electrostatic forces, so that the rods do not have to be touched; experiments on small specimens can be made, making possible a study of local properties of the wood; also the forces used are small and hence purely elastic. The frequencies of vibration are measured by means of frequency modulation. From the eigenfrequencies of the vibrating rods, the modulus of elasticity E, and from this the velocity of sound C, can be calculated. From the decay of successive vibrations one can determine the logarithmic decrement d, which is replaced in this investigation by the value $Q = \pi/d$, the so-called coefficient of resonance.

Experiments were made on 85 kinds of wood, with the rods oriented along and across the grain, from which it was found that both C and Q depend greatly on the angle of inclination with the grain, whereas the ratio C/Q is practically independent of this angle. Aging the wood seems to cause a considerable decrease of C and Q.

In these experiments, the influence of moisture was not considered, which may have considerably obscured the numerical results. F. Stüssi, Switzerland

Mechanics of Forming and Cutting

1279. C. Blazey, L. Broad, W. S. Gummer, and D. B. Thompson, "The flow of metal in tube extrusion," *J. Inst. Metals*, Dec. 1948, vol. 16, pp. 163–184.

Experiments on the distortion of billets due to extrusion through cylindrical orifices are described. The flow was studied by means of small pins inserted in the billets in the radial direction. These pins were drawn out during the flow, and from the observed patterns conclusions were drawn in regard to the surface friction in the skin of the billets.

In a nonlubricated container the portion of the billet near the surface did not move much relatively to the surface. The oxide layer on copper acted as a lubricant. When the copper was prevented from oxidation the billet flowed like brass, in the same manner as nonlubricated billets. A. Nadai, USA

Hydraulics; Cavitation; Transport

(See also Rev. 1320)

1280. Gernando Petracchi, "On the interpretation of the corrosion process by cavitation" (in Italian), *Metallurgia ital.*, 1949, vol. 41, no. 1, pp. 1–6.

After summarizing some former experiments and theories of cavitation, the author makes the supposition that galvanic effects are a major cause of cavitation damage. To prove this, he determines first the endurance strength of five different bronzes, brass, and one iron specimen, and makes fatigue tests by the rotating-beam method in air and sea water with the specimen acting as cathode or anode in an electric circuit. He finds that cathodic protection reinvests the specimen in sea water with the strength it had in air, and in some cases actually improves it.

Next, he applies cathodic protection to specimens in a special venturi-type test section under cavitation. He gets a similar damage and finds that if sufficient current is passed through the circuit, with the specimen as cathode, loss of weight is arrested.

He theorizes that repeated stressing of the specimen causes damage through a piezoelectric effect which results in corrosion due to circulating currents. Counterflow of current by application of negative potential to the specimen reduces the effect. This theory, while promising in the case of corrosive liquids, contradicts some previous research demonstrating mechanical action to be sufficient to cause cavitation damage in a noncorrosive liquid or on a nonmetallic probe. Aladar Hollander, USA

1281. Alessandro Veronese, "Experimental measurements on free discharges" (in Italian), *Energia Elett.*, Dec. 1948, vol. 25, pp. 638–641.

The author studies the flow in the zone immediately preceding the free overfall of (a) a broad-crested weir, (b) a channel under

torrential conditions, and (c) a pipe. In all these cases he gives experimental readings of the velocity profiles and of the pressure distribution in various sections.

He also gives an elementary theory based on the hypothesis of a hydrostatic pressure distribution in an upstream section and a constant pressure at the exit section; use is made of the constancy of the discharge or of the momentum theorem in neglecting the slope of the flow lines.

The tests with the channel and the pipe represent a new contribution when compared with the previous work of Rouse [*Verteilung der hydraulischen Energie bei einem lotrechten Absturz*, Oldenbourg, Berlin, 1933]; a more refined theory is given by Paderi [*Sulla chiamata di sbocco*, Pacini Mariotti, Pisa, 1948] and Jaeger [*Houille blanche*, Nov.-Dec. 1948]. A. Craya, France

1282. Antonio Fiorenzi, "On the direct and absolute measurement of weight, volume and heat in fluids" (in Italian), *Termotecnica*, Dec. 1948, vol. 2, pp. 397–401.

The author describes the principle of a device which, when combined with a Venturi or a similar instrument, will automatically furnish readings corrected for deviations of the temperature T and pressure P from their calibration values. With fluids used for heat-transfer purposes, it will read the heat transferred.

Eliminating temperature and pressure measurements, this device, utilizing changes in electric resistivity, will compensate for deviations of P and T under limited conditions specified by the author. L. Escande, France

Incompressible Flow: Laminar; Viscous

(See also Revs. **1223, 1230, 1248, 1297, 1301, 1302, 1303, 1305, 1307, 1328, 1336, 1338, 1341**)

1283. M. Manarini, "On the equations of the dynamics of perfect fluids" (in Italian), *Boll. Un. mat. ital.*, Aug. 1948, ser. 3, vol. 3, pp. 111–114.

An identity relating the acceleration vector in a continuous medium to the momentum transfer tensor pv^iv^j was noticed by Greenhill ["Hydromechanics," *Encyclopaedia Britannica*, 9th ed., 1875]. Employing this identity and introducing a four-dimensional treatment, von Laue [see Tolman, *Relativity, Thermodynamics, and Cosmology*, Oxford, 1934, pp. 36–38] constructed an energy-momentum tensor T^{ij} such that Cauchy's equations of motion and the d'Alembert-Euler continuity equation, suitably corrected for relativistic effects, assume the form $T^{ij}{}_{,j} + \rho f^i = 0$, whence momentum theorems follow immediately. These results remain valid a fortiori in the special case when relativistic effects are neglected and when the stress reduces to a pure pressure. Pailloux [*C. R. Acad. Sci. Paris*, 1947, vol. 225, pp. 1122–1124], without mentioning prior work on the subject, derived ab ovo the equations appropriate to this special case. The present author derives Pailloux's results in dyadic notation.

C. A. Truesdell, USA

Courtesy of Mathematical Reviews

1284. R. Westberg, "On the harmonic and bi-harmonic problems of a region bounded by a circle and two parallel lines" (in English), *Acta Polyt.*, 1948, no. 18, pp. 5–66.

This paper gives a detailed and somewhat complex mathematical treatment of the flow around a circular cylinder placed somewhere between two parallel planes. Both viscous and nonviscous flows are considered, while compressibility is neglected. The problem of nonviscous flow is first broken up into two auxiliary problems each of which has simpler boundary conditions than the complete problem. The velocity components are derived from a potential for each of the auxiliary problems. The potential, of course, satisfies the harmonic differential equation. The viscous flow solution is based upon Stokes' approximation for the hydrodynamic equations for slow motion, so that the inertia of the fluid is neglected. Therefore the stream function from which the velocity is derived satisfies the biharmonic differential equation. All solutions are obtained in terms of infinite series in the complex domain, each term of the series satisfying some of the boundary conditions and the governing differential equation. Convergence and some properties of all series are thoroughly studied.

Bruno A. Boley, USA

1285. R. Paul Harrington and Paul A. Libby, "The shear flow of a perfect fluid about a circular cylinder near a rectilinear boundary," *Reissner Anniv. Vol.*, J. W. Edwards, Ann Arbor, 1949, pp. 37–42.

A two-dimensional flow of a perfect fluid is considered. The center of a circular cylinder of radius a is located at a distance h above the x-axis, which is considered to be a solid boundary. The undisturbed flow velocity U is described by $U = by$. It is required to determine the flow about the cylinder.

The stream function ψ satisfies the equation $\Delta^2\psi = b$, is identically zero along the x-axis, has a constant value around the circular cylinder, and at infinity reduces to the stream function for the undisturbed flow.

The authors let $\psi = \psi_0 + \psi_1$, where $\Delta^2\psi_0 = b$ and $\Delta^2\psi_1 = 0$, ψ_0 representing the stream function of the undisturbed shear flow. ψ_1 must satisfy Laplace's equation and the boundary conditions that ψ_1 vanishes at infinity throughout the region and along the x-axis, and that $\partial\psi_0/\partial u = -\partial\psi_1/\partial u$ for $t = t_0$ (circular cylinder). Here u and t are the dipolar coordinates relative to the poles $(0, c)$ and $(0, -c)$, $c^2 = h^2 - a^2$. The authors show that $\psi_0 = \frac{1}{2}bc^2 \sinh^2 t/(\cosh t - \cos u)^2$ and

$$\psi_1 = -\pi^{-1}bc^2 \sinh^2 t_0 \sum_1^\infty \sinh nt \cos nu \, (\sinh nt_0)^{-1} \int_0^\pi \frac{\cos n\xi d\xi}{(\cosh t_0 - \cos \xi)^2}.$$

L. J. Tison, Belgium

1286. E. G. Richardson, "The impact of a solid on a liquid surface," *Proc. phys. Soc. Lond.*, Oct. 1948, vol. 61, pp. 352–367.

This paper is concerned with applications to water-entry ballistics and, to a lesser extent, to the impact of seaplanes on landing. The paper is divided in two parts.

Part I consists of a study of the motion of a solid sphere when it strikes a liquid surface vertically. From photographs the field of flow round the sphere is deduced, and it appears that the shape of the air cavity formed can be explained in terms of potential flow. The air pressure developed in the cavity during the underwater trajectory is measured and found to involve reverberations in the cavity of large amplitude. The resistance experienced by the sphere in this stage is also measured. It is shown that the forces scale on a Froude basis.

In part II some methods of calculating the impact force on the sphere on entry into the liquid are discussed. With the aid of small-scale experiments the impact forces on a sphere and other forms of projectile are determined and compared with the theoretical results. The mechanism of ricochet and the form of cavity associated with entry at glancing angles are also discussed.

R. J. Legger, Holland

1287. G. Temple and Jennifer Yarwood, "Two-dimensional aerofoils in ducts with uniform velocity distribution," *Rep. Memo. aero. Res. Counc. Lond.*, no. 2090, Feb. 1944 (issued in 1948), 17 pp.

The purpose of this investigation is to determine the exact form of two-dimensional airfoils and ducts which have uniform velocity distributions over the greater part of their surfaces. The discontinuities which would exist at the leading and trailing edges if the velocities were uniform over the whole of the upper and lower surfaces, respectively, are avoided by adding a short beak and tail, over which the velocities are equalized without any discontinuity. The calculation is completed for the case of the symmetric duct and it is found that the beak and tail are of negligible dimensions.

The mathematics required to determine the form of the constant-velocity airfoil and duct by the methods of conformal representation is included, and it is shown that the usual methods of free streamline theory can be simplified and shortened.

Nicholas Di Pinto, USA

1288. F. W. S. Locke, Jr., "An empirical study of low aspect ratio lifting surfaces with particular regard to planing craft," *J. aero. Sci.*, Mar. 1949, vol. 16, pp. 184–188.

An empirical study is presented of the lift coefficients of low-aspect-ratio wings, planing surfaces and hulls. On a basis of low-aspect-ratio wing data published by Winters and Zimmerman, and planing data published by Sottorf and from the files of the Stevens Towing Tank, it is concluded that the lift coefficient may be related to the angle of attack α by,

$$C_L = \eta K \alpha^n$$

where K and n are functions of the aspect ratio and are given in the paper in graph form. η is a function of the type of lifting surface and, if assumed to be unity for airfoils, becomes 2/3 for hulls and 1/2 for flat-bottom planing surfaces.

R. L. Bisplinghoff, USA

Compressible Flow, Gas Dynamics

(See also Revs. 1283, 1301, 1302, 1304, 1313, 1323)

1289. M. Manarini, "On the D'Alembert and Brillouin paradoxes in fluid dynamics" (in Italian), *R. C. Accad. Lincei*, Apr. 1948, ser. 8, vol. 4, sem. 1, pp. 427–433.

Consider a rigid body moving uniformly on a helix through inviscid compressible fluid, which extends to infinity in all directions. It is proved that d'Alembert's paradox, absence of direct resistance, subsists even when the fluid contains vortex sheets which do not extend to infinity. The proof is simple and is based upon a vectorial application of Gauss' theorem, together with the hypothesis that the mean values of $\rho v r^3$ and $(p - p_0)r^2$, over a sphere of radius r, tend to zero as $r \to \infty$. In like manner it is proved that Brillouin's paradox holds, that the fluid must contain regions of negative pressure. The author concludes that neither paradox can be avoided in the absence of an infinite wake.

L. M. Milne-Thomson, England

Courtesy of Mathematical Reviews

1290. J. W. Craggs, "The breakdown of the hodograph transformation for irrotational compressible fluid flow in two dimensions," *Proc. Camb. phil. Soc.*, July 1948, vol. 44, pp. 360–379.

In the first half of the paper the author considers singularities of general mappings between arbitrary (x, y) and (u, v) planes such that u and v are analytic functions of x and y. A careful, almost exhaustive, mathematical classification of the various possible subcases of singular points and curves of order one (at which the Jacobian $J = \partial(u, v)/\partial(x, y) = 0$, while J_x or $J_y \neq 0$) and of order two (at which $J = J_x = J_y = 0$ while at least one of the second partial derivatives of J does not vanish) is presented together with graphical illustration of the mapping of curves passing through the singularities.

In the second half of the paper the mappings are specialized to those between the physical and hodograph planes (and their inverses) corresponding to possible irrotational compressible fluid flows in two dimensions. With the help of the general theorems established in the first part, the possible singularities of the flow mappings are described. Special attention is paid to the features distinguishing the limiting lines (ψ_q, ψ_θ remain finite; limiting line is an envelope of Mach lines) from the "branch lines" (ψ_q, $\psi_\theta \to \infty$; line is a Mach line). It is shown that a branch line (also known as transition line) must occur in any transonic flow through an unbent nozzle, thereby causing a breakdown of any series representations. The results of the study overlap with those obtained independently in Courant-Friedrichs, *Supersonic Flow and Shock Waves*, pp. 63–69 and 257–259.

M. V. Morkovin, USA

1291. Antonio Ferri, "Elements of aerodynamics of supersonic flows," Macmillan Co., New York, 1949. Cloth, 9.4 × 6.25 in., 434 pp., 248 figs., $10.

As the author states in the preface, the aerodynamics of supersonic flow is considered in elementary form, from the aeronautical point of view, in the present book. The most helpful analytical and calculus methods (small disturbances, characteristics theory) are presented and illustrated by a detailed discussion of a great number of examples. It is thus attempted to equip the reader for the practical use of these methods. A more concise and systematic presentation of the basic mathematical concepts, especially for the characteristics theory, would make some parts of the book easier to read and more agreeable to the mathematically inclined.

After a brief survey of fundamental aerodynamic and thermodynamic concepts in chapter 1, the two-dimensional potential flow and rotational flow is treated in the usual manner in chapters 2 to 5. Measurement methods (for instance interferometric methods for two-dimensional and axisymmetric flow, shadow and schlieren apparatus) are presented in chapter 6. Chapters 7 to 9 deal with practical applications (detailed discussion of supersonic profiles, two-dimensional diffusers and effusers).

Chapters 10 to 15, dealing with three-dimensional phenomena, are doubtless the most interesting part of the book. After general considerations on three-dimensional linearized potential flow, it continues with the presentation of the Kármán-Tsien linear theory for bodies of revolution, a detailed study of nonlinear conical flow (Busemann's hodograph method generalized for yawing cones) a discussion of nonlinear potential and rotational flow around bodies of revolution with and without yaw (characteristics method for two and three independent variables, semilinear method for small angles of attack). The author fails to emphasize that, for the supersonic conical flow problem, one has to distinguish between hyperbolic and elliptic fields, and that the neglect of vorticity in the case of yawing bodies is not justified. Finally, the linear theory of sinks, sources and doublets distributions for finite wings is treated and followed by a detailed discussion of pressure drag, lift and induced drag for triangular and rectangular wings, sweptback wings (with supersonic or subsonic leading edges) and other plan forms. The classical Busemann method for linearized conical flow, used successfully by W. D. Hayes, H. J. Stewart and others, is mentioned but not presented.

The book will be very helpful to the engineer interested in

supersonic problems. For a new edition of the book, the reviewer suggests the addition of a chapter on transonic and hypersonic phenomena (Kármán-Tsien's similarity laws, Sauer's simplified characteristics method for yawing bodies of revolution in hypersonic flow). Robert Sauer, Germany

1292. **Jacques Nicolas, "On a graphical method for two-dimensional supersonic flows"** (in French), *Off. nat. Étud. Rech. aéro. Rep.*, no. 12, 1948, 10 pp. and 2 charts.

As is well known, a supersonic flow with weak shocks can be determined approximately by Busemann's method of characteristics if one treats the shocks as compression waves which produce a larger change of state. In order to estimate the error occurring with such a construction, the author analyzes the conditions which determine a shock, and then develops formulas for the change of the flow past the shock if the conditions upstream of the flow are changed by a Mach wave arriving either from the right or from the left boundary of the flow. Gottfried Guderley, USA

1293. **A. Robinson, "On source and vortex distributions in the linearized theory of steady supersonic flow,"** *Quart. J. Mech. appl. Math.*, Dec. 1948, vol. 1, pp. 408–432.

The elementary "source" solution of the linearized velocity-potential equation for supersonic flow has been used extensively recently in the construction of linearized supersonic flows about finite wings [Schlichting, *Luftfahrtforsch.*, 1936, vol. 13, pp 320–335; Jones, *Nat. Adv. Comm. Aero. tech. Note*, no. 1107, 1946; Puckett, *J. aero. Sci.*, 1946, vol. 13, pp. 475–484; and others].

Application of this solution is hampered by the fact that it becomes infinite not only at the source itself but also at all points of its characteristic (Mach) cone. Such difficulties can be overcome by using Hadamard's concept of the finite part of an improper integral [Hadamard, *Lectures on Cauchy's Problem*, Yale Univ. Press, 1923].

The author discusses the finite part of a somewhat more general integral than Hadamard, defines a "hyperbolic nabla of index β," namely, $\nabla h\beta = (-\beta^2\, \partial/\partial x,\ \partial/\partial y,\ \partial/\partial z)$, and hence a hyperbolic gradient and divergence. He establishes the divergence theorem and Stokes theorem for the finite part of an integral. These tools are applied to a discussion of source and vortex distributions in space. In particular, Gauss' theorem on the strength of sources within a given surface is established, the continuity of the potential derivatives across a surface distribution of sources is discussed, and the flow field for the horseshoe vortex is found. William H. Pell, USA

1294. **T. Y. Thomas, "Calculation of the curvatures of attached shock waves,"** *J. Math. Phys.*, Jan. 1949, vol. 27, pp. 279–297.

A formula has been derived [*J. Math. Phys.*, vol. 26, pp. 62–68] for the ratio of the curvature of a shock line to the curvature of the stream line immediately behind the shock line. This paper considers a pointed body of revolution placed in a uniform supersonic flow of gas for which the pressure, density, and velocity vector are constant. The direction of flow coincides with the axis of the obstacle. The previously derived formula is employed to calculate the angle of inclination of the attached shock wave at its point of attachment to the obstacle in terms of the Mach number of the undisturbed flow and the angle of inclination of the obstacle at its vertex. The ratio of the heat capacities is taken to be 1.405. The results of the calculations are exhibited in graphical and tabular form. Stuart R. Brinkley, Jr., USA

1295. **M. J. Lighthill, "The position of the shock-wave in certain aerodynamic problems,"** *Quart. J. Mech. appl. Math.*, Sept. 1948, vol. 1, pp. 309–318.

The velocity of the shock wave preceding a cylinder or sphere expanding slowly and uniformly into still air is calculated. The pressure on the disturbing surface is obtained from the nonlinearized Bernoulli equation, although linearized theory is found to be adequate for the velocity field. Analytical comparison is made with Riemann's case of a moving plane. A numerical check shows reasonable agreement with Taylor's exact numerical solution for the spherical case.

A similar analysis is made of the steady supersonic flow over an unyawed cone of small vertex angle, yielding the shape of the conical shock wave. Comparison of results with the exact numerical solution of Taylor-Maccoll for a cone of 20-deg vertex angle shows good agreement. The pressure on the surface of the cone is again obtained from the nonlinearized Bernoulli equation. Comparison is made with Meyer's case of flow over a two-dimensional wedge.

The similarities between the above two classes of problem are indicated. It is pointed out that the analysis is mathematically rigorous, although directed to asymptotic results.

It is deduced that the shock wave near the convex conical tip of a slender body of revolution will have the same strength as if the body were a cone of the same vertex angle, while the remainder of the shock will be weaker than in the conical case.

W. G. Cornell, USA

1296. **Gottfried Guderley, "Considerations of the structure of mixed subsonic-supersonic flow patterns,"** *Hdqtrs. Air Mat. Comm. Dayton tech. Rep.*, no. F-TR-2168-ND, Oct. 1947, 92 pp.

This paper shows how the general properties of compressible-flow patterns in physical and hodograph planes can be used to study mixed subsonic-supersonic patterns when the boundary conditions can be simply formulated in the hodograph plane. Shock waves following uniform flow can be approximately included in this category if the deviations from sonic velocity are small. Then Tricomi's existence and unicity theorem for regular solutions of the simplest kind of mixed-type equation may be applied to the hodograph equation and indicates that the curve in the hodograph plane along which the boundary conditions are given must, for regular solutions in the mixed case, present a supersonic gap. By methodical use of the mentioned properties the author constructs qualitative solutions in the following cases:

(1) Free jet, including discussion of the transition from the subsonic Chaplygin case to the fully developed supersonic case.

(2) Wedge in a supersonic stream, including discussion of the continuous transition from supersonic flow with attached shock to mixed flow with detached shock, and the meaning of Crocco's point (the point of the shock polar where the streamlines leaving a curved shock have no initial curvature) with regard to the separation of the regions of shocks with zero and infinite initial curvature.

(3) Wedge with curved sides near Crocco's point. Crocco suggested that the detachment of the shock could take place at this point. It is shown that the detachment begins only at the point of maximum deflection, but the initial curvature of the shock becomes infinite between this point and Crocco's point.

(4) Supersonic jet with counterpressure, with a discussion of the transition from the supersonic solution with crossed shocks to solutions with λ-shocks. A new type of λ-shock is indicated in whose branch point a subsonic region ends and a Meyer expansion gives rise to an adjacent supersonic closed region. This is in contradiction with the necessity of a supersonic gap for regular solutions. Hence a singularity must arise. The author does not

explain clearly how the singularity is compatible with the physical existence of the flow.

(5) Flow past a double wedge in supersonic stream, including the great variety of cases arising from the variation of stream conditions and of angle and length of both wedges.

(6) Flow over a wedge in a tunnel, with a discussion of the effect of the relative dimensions, rate of mass flow and counterpressure. In both cases the new type of λ-shock may be present and the "strong solution" for the shock may appear as the result of the coalescence of the "weak solution" and of a normal shock. Finally the approximate quantitative behavior of solutions near particular points are derived. As the author points out, important results of this study are that the transition to mixed flows is continuous, and that the solutions and particularly the position of shocks are determined by well-defined boundary-value problems.

Luigi Crocco, Italy

1297. Gottfried Guderley, "Perturbations in plane, axisymmetric, parallel jets in the transonic and supersonic range" (in German), *Z. angew. Math. Mech.*, Oct. 1947, vol. 25/27, pp. 190–195.

The author has investigated the effect of the propagation of weak disturbances in a parallel two-dimensional or axisymmetric jet of sonic or supersonic velocity.

A system of rectangular coordinates ξ', η' is used, where η' lies along a Mach line of the parallel flow. When $\xi' < 0$ the flow is assumed parallel, while in the region $\xi' > 0$ the flow is affected by a specified kind of disturbance. The velocity potential of the parallel flow, $\xi' < 0$, is written:

$$\Phi = W_0 l(\xi \sin \alpha_0 + \eta \cos \alpha_0)$$

where ξ, $\eta = \xi'/l$, η'/l, respectively, and l is a reference length. W_0 and α_0 are respectively the velocity and Mach angle of the parallel flow. For values of $\xi' > 0$ the velocity potential becomes:

$$\Phi = W_0 l(\xi \sin \alpha_0 + \eta \cos \alpha_0 + f(\eta)\xi^m).$$

The term $f(\eta)\xi^m$ represents the potential of the disturbance, $f(\eta)$ its strength and the index m its character. The expression for Φ is substituted into the general equation for the velocity potential and only the lowest powers of ξ in each term are retained.

When the parallel flow is supersonic it is shown that a disturbance for which $m < 2$ cannot exist. When $m = 2$ the two-dimensional flow experiences an expansion when $f(\eta)$ is positive, and a compression when $f(\eta)$ is negative. When $m \geqslant 2$ the axisymmetric flow experiences a compression, but the solution has a singularity of the order $\eta^{-1/2}$ on the axis of symmetry.

It is shown that a disturbance for which $m < 3$ is not possible when the parallel flow is sonic. When $m \geqslant 3$ the flow becomes supersonic when $f(\eta)$ is positive, and subsonic when $f(\eta)$ is negative. In the latter case the velocity falls along the axis from sonic to zero velocity in a finite length. The author considers this result to be important since it implies that when the velocity of flow is reversed, a parallel sonic jet is obtained from a subsonic flow in a finite length.

G. M. Lilley, England

1298. Fritz K. Feldmann, "Investigation of symmetric wing profiles at high subsonic velocities in a closed wind tunnel" (in German), thesis, Eidgenössische Technische Hochschule, Zurich, 1948, 70 pp.

The paper describes an experimental investigation of six symmetric-section airfoils of constant rectangular plan form, tested up to high subsonic Mach numbers at a constant Reynolds number of 0.5×10^6. The measurements of lift, drag and pitching moment were made on a special three-component balance. In addition the flow was observed with a schlieren mirror system.

A zero pressure gradient along the tunnel axis was obtained by variations of the fillet size to allow for the growth of the boundary layer and the blockage effect of the model supports. In this way Mach numbers up to 0.965 were obtained at incidences up to 7.0 deg. The experimental readings have been corrected for wind-tunnel interference but *not* for blockage. The problem of blockage corrections to the free-stream Mach number was investigated experimentally and the results were compared with those given by A. Thom ["Blockage corrections in a closed high-speed tunnel," R. and M. 2033, 1943]. Large discrepancies were obtained between the two methods above the critical Mach number.

The results show quite good agreement with the Prandtl-Glauert rule up to the critical Mach number. The importance of thickness is clearly shown, and for a 12 per cent thick section a reversal of lift was obtained in a Mach number range of 0.86 to 0.88.

G. M. Lilley, England

1299. Joseph B. Keller, "On the solution of the Boltzmann equation for rarefied gases," *Commun. appl. Math.*, Sept. 1948, vol. 1, pp. 275–285.

The general Boltzmann equation for the distribution function is studied with the view of obtaining solutions appropriate to the determination of the drag of bodies in rarefied gases. The solutions of the equation have been studied extensively for bodies very large or very small compared to the mean free path. The author attempts solutions in which the body is smaller than, but comparable to, the mean free path. By a perturbation method a power series expansion for the solution is obtained. The coefficients of this power series are obtained in succession, each depending upon all previous coefficients. The first coefficient is the solution for free molecular flow. If this series is convergent, it can be made to fit the initial and boundary conditions of the problem. These boundary conditions are discussed in terms of a probability function for the velocity distribution of reflected molecules, and a probability function for the velocity distribution of particles spontaneously leaving the surface. The author then gives the probability functions appropriate to specular, diffuse and combined reflections. Although the author does not discuss the matter, it would be presumed that the probability for spontaneous emission of particles would cover the case of molecular absorption and re-evaporation. Application is made to free expansion in one dimension. This latter solution is compared with the exact gas-dynamical solution.

Howard W. Emmons, USA

1300. J. M. Burgers, "Cases of motion in a gas with noncolliding molecules" (in English), *Proc. kon. Ned. Akad. Wet.*, June 1947, vol. 50, pp. 573–583.

The problem considered is to calculate the velocity distribution function of molecules between two parallel walls, of infinite extent, or of molecules on one side of a single wall under the assumptions that (1) the velocity of the wall V is normal to the plane of the wall in the direction of the x-axis; (2) the initial distribution before the motion of the wall is specified; (3) the internal energy of the molecules is zero; (4) the kinetic energy relative to the wall carried by the molecules is not changed by collision with the wall; (5) the distribution function for reflected molecules from the wall is given by $F(t; u, v, w) = A(h/\pi)^{3/2} \exp\{-h[(u - V^2) + v^2 + w^2]\}$ where v and w can take all values while $u = V$ for the wall on the left of the gas; (6) the collisions among the molecules can be neglected. It is seen that the only conditions for determining the distribution function are (2) and (4).

Two cases are treated in some detail: (*a*) The case of a single wall moving with high velocity and zero initial molecular velocity.

Here considerable deviation from the Maxwellian distribution is found. (*b*) The expansion of rarefied gas between two symmetrically receding walls. Here only the first steps of solution could be taken for the special case where the velocity of the walls is small compared to that of the molecules. Then the temperature of the gas is found to decrease in a way predicted by Poisson's law for adiabatic change of state.

H. S. Tsien, USA

Turbulence, Boundary Layer, etc.

(See also Rev. 1230)

1301. **N. A. V. Piercy and L. G. Whitehead, "Boundary layer calculations,"** *Aircr. Engng.*, Jan. 1949, vol. 21, pp. 17–19.

The authors show two simplifications of the method for solving the equation of the boundary layer, already suggested by them in a previous paper [*Aircr. Engng.*, Dec. 1949, vol. 20, pp. 354–359]. There the problem was reduced to the integration of two differential equations for the determination of two factors K_1, K_2 which: (a) are functions only of the coordinate η measured along the stream lines of irrotational flow and of $\phi = 2 - 4(\eta/h)(\partial h/\partial\eta)$, in which h is the velocity outside the boundary layer; (b) reduce, as does ϕ, to constants for $h = K\eta^m$.

The first simplification suggested is the reduction of the problem to the integration of only one differential equation, obtained by setting $K_2 = K_1^2$. To this end, two other functions from the previous paper, f_1 and f_2, which also express the velocity in the boundary layer, are modified in such a way that this substitution for K_2 does not reduce the accuracy when the velocity outside the boundary layer varies according to the law $h = K\eta^m$. The authors indicate therefore how the differential equation for K_1 can then be solved by a method of successive approximations, or by a step-by-step method.

The second simplification, even more drastic than the first, consists essentially in assuming for K_1 the values which are obtained when the function ϕ, which is generally variable, is replaced by an appropriate average value.

Applications to problems for which the exact solution is known (elliptic cylinder, linearly retarded and accelerated flow) show that the approximate results reached by both methods are good, except in the immediate vicinity of the breakaway. The authors apply their simplified method to establish the effect on the boundary layer of a space fluctuation of velocity, a problem which has already been studied by Dryden using Polhausen's method.

Carlo Ferrari, Italy

1302. **L. G. Whitehead, "An integral relationship for boundary layer flow,"** *Aircr. Engng.*, Jan. 1949, vol. 21, pp. 14–16.

By double integration of the boundary-layer equation, the author determines an integral relationship which: (a) can be applied, in place of that of von Kármán, to obtain an approximate solution with velocity profiles across the boundary layer depending on one parameter; (b) can be added to that of von Kármán to obtain velocity profiles across the boundary layer depending on two parameters.

Applications made by the author to motion problems for which the exact solution is known (flat plane; flows in which the velocity varies according to the law $V = ks^n$; linearly retarded flow) show that method (a) yields a better approximation to the coefficient of friction, while the velocity profile is obtained with a slightly inferior accuracy, and that method (b) yields a better approximation to velocity and an even better approximation to the coefficient of friction.

Carlo Ferrari, Italy

1303. **H. Schlichting, "An approximate method for calculation of the laminar boundary layer with suction for bodies of arbitrary shape,"** *Nat. adv. Comm. Aero. tech. Memo.*, no. 1216, Mar. 1949, pp. 1–85 (transl. from *Aero. Inst. T. H. Braunschweig Ber.*, no. 43/13, June 1943).

This paper is a clear presentation of the technique for applying the von Kármán integral relation to laminar boundary-layer flows with both pressure gradient and surface suction. After the general relations have been deduced and the significant parameters have been identified, solutions of flat plate and stagnation flows, both with and without suction, are used as simple illustrations. Then the more general examples of boundary layers on a circular cylinder and a Joukovsky profile, each with homogeneous suction, are worked out. Numerical tables are included.

Stanley Corrsin, USA

1304. **Neal Tetervin, "Remarks concerning the behavior of the laminar boundary layer in compressible flows,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1805, Jan. 1949, 20 pp.

The nondimensional equations of motion for steady compressible flow in the boundary layer of two-dimensional and axially symmetric bodies are given with the usual approximations. When the dimensionless boundary conditions on velocity and temperature are independent of the Reynolds number, the analysis shows that at a fixed Mach number the location of the separation point is also independent of the Reynolds number and can occur only in an adverse pressure gradient. Under the same conditions the boundary-layer thickness and skin-friction coefficient are inversely proportional to the square root of the Reynolds number.

Arnold M. Kuethe, USA

1305. **H. Görtler, "Influence of small waviness in the wall surface on the course of laminar boundary layers"** (in German), *Z. angew. Math. Mech.*, Nov.-Dec. 1947, vol. 25/27, pp. 233–244; Jan. 1948, vol. 28, pp. 13–22.

The paper treats the calculation problem of the velocity profile of laminar boundary layers along walls with small waviness. Initially the waviness is introduced as a periodic disturbance with small amplitude in the velocity at the outer edge of the boundary layer, characterized by a wave length λ, assumed small, amplitude ϵ, expressed in terms of the local free-stream velocity U_m. Under restriction to first-order terms the equations for the disturbance profile are established in terms of the undisturbed velocity profile, ϵ, λ and a nondimensional parameter $\sigma = (\nu\lambda)/(U_m 2\pi\delta_0^2)$ (δ_0 = boundary layer thickness), representing the viscosity.

For the case of the Blasius flow, at a distance L from the leading edge, if $\sigma = \lambda/(2\pi L)$ is small, the equations are solved numerically for the cases $\sigma = 0.01$, $\epsilon = 0.01$ and 0.005, by power series near the wall and asymptotic expressions at large distances, linked together at a suitable point. Special attention is given to checking the numerical accuracy.

Afterwards, using a concept given by Prandtl, an estimate is given of the wall waviness corresponding to the assumed value of free-stream velocity fluctuations. The "effective" waviness at the outer edge of the boundary layer appears to be much smaller than the actual waviness of the wall, whose influence is restricted to a small "disturbance boundary layer." The thickness of the latter, expressed in the total boundary-layer thickness is proportional to $(\lambda/L)^{1/3}$. In the case of sufficiently small δ_s the Blasius profile in the region near the wall is approximated by a straight line, which yields an analytic expression of the disturbance profile in Hankel functions, valid only for $\sigma \leq 0.01$. A criterion for local separation is found: $\epsilon > 0.168\,\sigma^{2/3}$.

In this case again the actual shape of the wall is calculated, cor-

responding to a prescribed value of ϵ. This is done by means of estimates of the asymptotic behavior of the disturbance profile and the separation criterion is expressed in terms of the actual wall waviness. It should be remarked, that the flow actually separates only after a large number of waves. Initially, only local regions of practically dead water in the valleys of the wall are formed. The results are compared with unpublished results from numerical computations by Quick and Schröder.

Finally the total resistance of the undulating wall is found to be equal to the resistance of the flat plate. R. Timman, Holland

1306. Tsutomu Maekawa and Shoichi Atsumi, "Transition caused by laminar flow separation" (in Japanese), *J. Soc. appl. Mech. Japan*, Nov. 1948, vol. 1, pp. 187–192.

The authors report on the results of wind-tunnel experiments on a bent profile, which is composed of two flat plates making an (adjustable) obtuse angle. The laminar boundary layer along the forward plate separates at the bend and, in certain circumstances, reattaches itself to the rear plate, forming a turbulent layer. The experimental results are satisfactorily explained by A. E. von Doenhoff's consideration [*Nat. adv. Comm. Aero. tech. Note*, no. 639, 1938] that the separated layer becomes unstable at a certain Reynolds number, and then spreads in the form of a turbulent jet, although, after the steady state is reached, it is really the attraction of the back-flow vortex lying downstream of the separation point that drives the separated layer onward. The stability limit of the separated layer is given by a Reynolds number 25,000, which is independent of the stream turbulence. The instability does not occur, however, when the Reynolds number of the boundary layer thickness at the separation is less than 1200. The angle of spread of the turbulent jet increases as the stream turbulence increases. If the jet reaches the downstream surface at a point too far from the separation, the boundary layer cannot be completely established, thus resulting in the subsequent separation and reattachment. This is explained by supposing that the vorticity produced at the separation point is no longer sufficient to maintain the back-flow vortex. Itiro Tani, Japan

1307. J. R. Weske, "Investigations of the flow in curved ducts at large Reynolds numbers," *J. appl. Mech.*, Dec. 1948, vol. 15, pp. 344–348.

The turbulent flow in curved ducts is considered. Qualitative reasoning shows the flow to be made up of three regions: (1) the main body of fluid wherein the axial velocity component is the largest; (2) the region near the wall in which the velocity component tangential to the wall is of the same order of magnitude as the axial component; and (3) the region nearest the center of curvature of the bend where the fluid has large scale eddies.

The flow in region (2) above is analyzed according to approximations of the boundary-layer type, for two-dimensional flow in a cross section normal to the bend axis. Assuming that the pressure drop along the bend due to the secondary flow is proportional to the kinetic energy of the fluid transported peripherally, and assuming that the effect of the inlet velocity distribution outweighs that of the equilibrium secondary flow, an expression is derived for this pressure drop as a function of the ratio of duct radius to radius of curvature, the ratio of duct length to equivalent helical length of the secondary flow, and the ratio $(\theta + \delta_1)/r$, where θ and δ_1 are respectively the momentum and displacement thicknesses of the entrance velocity profile and r is the duct radius.

Experimental verification of the character of the dependence of secondary pressure drop on duct radius ratio and $(\theta + \delta_1)/r$ is presented. Further experiments are described which support the validity of the assumption that the secondary pressure drop is proportional to the kinetic energy of the fluid transported peripherally. W. G. Cornell, USA

Aerodynamics of Flight; Wind Forces

(See also Revs. 1239, 1287, 1288, 1303, 1313)

1308. G. A. Crocco, "Kinematic parameters of a flight path" (in Italian), *Aerotecnica*, Feb. 15, 1949, vol. 29, pp. 4–11.

In order to further the understanding of the airplane motion on a path of double curvature the author introduces new coordinates, using the well-known definitions and relations of differential geometry. He establishes many useful kinematic relations between these coordinates and those usually applied, which—he hopes—may help to study instrumental flight.

I. Flügge-Lotz, USA

1309. J. G. Lowry and L. E. Schneiter, "Estimation of effectiveness of flap-type controls on sweptback wings," *Nat. adv. Comm. Aero. tech. Note*, no. 1674, Aug. 1948, 23 pp.

Two methods of estimating the lift, pitching moment, and rolling moment due to deflection of a flap-type control on a sweptback wing are given. These methods consider a rational spanwise load distribution due to the flap deflection, and the effects of sweepback, aspect ratio, and taper ratio. The results of estimates made by the proposed methods are compared to wind-tunnel tests, and good correlation is shown. Charts are included in the report which may be used to estimate control effectiveness.

W. O. Breuhaus, USA

1310. J. P. Uffen and A. D. Wood, "Interim report on measurements of landing loads on a ski plane," *Nat. res. Counc. Can. Mech. Engng. Rep.*, no. MM-210, 1948, 32 pp.

The paper describes a load-measuring pedestal, and presents data on landing loads taken in 31 landings made on an ice runway with a Noorduyn Norseman Mark VI airplane. The distribution of bending moment along the ski was also determined, and some data on the coefficient of friction are presented.

W. P. Welch, USA

1311. A. Lausetti, "Performance calculation for jet-propelled aircraft" (in Italian), *Aerotecnica*, Dec. 15, 1948, vol. 28, pp. 312–320.

A method is presented for estimating the level and climbing flight, and distance and duration performances for jet aircraft in subsonic flight. The knowledge of the polar diagrams for Mach numbers up to critical, and of thrust and consumption of turbojets in terms of the number of revolutions, of speed and height are needed. For these, some semiempirical diagrams are proposed, by means of which their determination from their values at zero level is simplified. The calculations are simple and easy.

The equations (29) and (30) of the paper are true only for a constant economy factor EV/k, but this does not affect the following conclusions. Gino Moretti, Argentina

1312. J. K. Zbrozek, "Stability and control of single rotor helicopter with hinged blades," *Aircr. Engng.*, Feb. 1949, vol. 21, pp. 32–38.

This is a clear summary, with a minimum of mathematics, of the problems of the dynamic stability and control of a helicopter with a single-hinged rotor blade. Among the topics discussed are: stability of the flapping motion of a blade in forward flight, rela-

tion between feathering and flapping for stability in forward flight, tilting of the no-feathering axis, dynamics of the longitudinal motion of a helicopter with fixed position of the no-feathering axis relative to the helicopter, and effect of various parameters including Lock's inertia number and hinge offset. The principle of the Bell stabilizing bar, with damper, for obtaining positive dynamic stability of the helicopter machine in hovering is described. The lateral behavior and the steady turn of a helicopter are briefly discussed. Numerical examples, with curves, are given.

Although this article contains little that is new, it serves to present a very good over-all view of the physical aspects of helicopter dynamics. A more complete list of references might have been desirable. Morris Morduchow, USA

Aeroelasticity (Flutter, Divergence, etc.)

(See also Rev. 1227)

1313. **P. I. Kooznetzov, "Wing profile vibrations in a supersonic flow"** (in Russian), *Notes Acad. Sci. USSR (Doklady Akad. Nauk SSSR)*, Jan. 21, 1949, vol. 64, pp. 301–304.

Oscillations of a thin airfoil in a supersonic stream are considered on the basis of the conventional perturbation theory. The Laplace-transform analysis is used and it is assumed (incorrectly) that both the velocity potential and its flow-direction derivative are continuous across the Mach wave emanating from the leading edge of the obstacle. The reviewer believes that this leads, in general, to incorrect results. George Carrier, USA

Propellers, Fans, Turbines, Pumps, etc.

(See also Revs. 1246, 1275, 1312)

1314. **A. D. S. Carter, "Three-dimensional-flow theories for axial compressors and turbines,"** *Proc. Instn. mech. Engrs.*, 1948, vol. 159, no. 41, pp. 255–268.

This lecture is a rather complete discussion of the flow in axial compressors and turbines. Among the topics discussed are: (1) Secondary flows and their relationship with vorticity due to change in circulation along the blade. (2) The inlet velocity profile in a cascade is shown to have a considerable effect on the amount of vorticity leaving the cascade, and therefore on the losses. (3) The effect of axial pressure gradient on secondary flow and efficiency. (4) Radial equilibrium. (5) The effect of tip leakage and the "moving-wall effect" on the passage flow with blading having tip clearance. (6) Boundary-layer displacement due to secondary flow and centrifugal force and its effect on stall and efficiency. (7) The thickening of the boundary layer at hub and tip due to accumulated secondary-flow effects caused by changes in velocity profile which lead to hub and tip stalling.

C. A. Meyer, USA

1315. **D. H. Mallinson and W. G. E. Lewis, "The part-load performance of various gas-turbine engine schemes,"** *Proc. Instn. mech. Engrs.*, 1948, vol. 159, no. 41, pp. 198–219.

The results of a very excellent study of the part-load performance of several gas-turbine arrangements are given. This lecture presents the calculated part-load performance of simple, open and closed, cross-compound, simple-compound, intercooled, regenerated and reheated gas turbines in various combinations. The calculations also cover various arrangements of the power take-off turbine. The calculations are based on different assumptions as to turbine- and compressor-flow characteristics and efficiencies, to show the effects of these assumptions.

Various operating conditions, such as constant tip temperature, constant mass flow or constant pressure-ratio are also covered by the calculations.

Although the number of types of cycles calculated represents only a small portion of the two hundred possible types, the results (27 figs.) indicate the guiding principles by which comparative part-load performance of the different types of gas turbines can be judged. C. A. Meyer, USA

1316. **W. H. Lindsey, "The development of the Armstrong Siddeley Mamba engine,"** *J. roy. aero. Soc.*, Feb. 1949, vol. 53, pp. 137–180.

This paper describes the development of a 1000-hp propeller-driving gas turbine. The work done on some of the components of the engine is covered in some detail including the axial compressor, a vaporizing-type combustion chamber, the propeller reduction gear and the controls. Some typical installations are also presented. Andrew A. Fejer, USA

1317. **Hayne Constant, "The prospects of land and marine gas turbines,"** *Proc. Instn. mech. Engrs.*, 1948, vol. 159, no. 41, pp. 191–197.

The paper deals with the present state of the practical development of the gas turbine, and reviews the economic and technical conditions required to make it a practical machine for land and sea applications. The technical problems involved are, above all, presented by the heat exchanger, of the stationary or rotary type, the filtration of the air before entering the compressor-inlet, and the combustion of cheap residual oils.

If these problems are solved, mechanical drive through fixed gears replacing the conventional heavy and expensive electrial drive will be the main advantage of the turbine-driven railway locomotive.

In the field of power stations, while burning coal internally is still impossible, the complex combination of gas turbine, gas producer, and utilization of the waste heat of the turbine, offers immediate advantages. In conjunction with a steam turbine fed from a waste-heat boiler, brake efficiencies of about 36 to 40 per cent can be obtained, while capital costs are estimated to be lower than for comparable steam stations.

In considering marine applications the author has no answer as to the type of turbine cycle to be adopted. High thermal efficiency on the one hand, and low bulk, weight, and cost on the other, are deciding factors. There may be some applications to highly rated Diesel engines, for instance for ocean-going ships.

In conclusion, the author forecasts a slow evolution of the gas turbine over the next two or three decades.

The reviewer believes that, in this connection, progress in high-temperature materials with improved creep and fatigue-resisting properties, as well as ingenious designs applicable to higher initial temperatures, are not to be underestimated. On the other hand several secondary problems cannot be overlooked; for instance, the important problem of transmission systems for ship-propulsion gas-turbine power plants. Robert Szewalski, Poland

1318. **Frank M. Lewis, "Propeller tunnel notes,"** *Trans. Soc. nav. Archit. mar. Engrs.*, 1947, vol. 55, no. 7, 12 pp.

Several aspects of testing marine propellers in water tunnels are discussed, with the major emphasis on investigating cavitation characteristics. The cavitation-testing procedure in use at the Massachusetts Institute of Technology Propeller Testing Tunnel

is outlined. Some experimental data are presented to show (1) the relation between tunnel tests and open-water (model-basin) tests, and (2) the relation between open-tunnel and closed-tunnel tests. The effects of air content on cavitation are discussed briefly. Experimental data are presented showing the effects of irregular approach of flow on propeller characteristics. Finally, there are presented several working charts showing the cavitating characteristics curves of a series of wide-bladed propellers designed by Gibbs and Cox for operation under cavitating conditions. Ascher H. Shapiro, USA

Flow and Flight Test Techniques

(See also Revs. 1282, 1298, 1318, 1338)

1319. Henry Girerd and Paul Guienne, "New static pressure gages for aerodynamic measurements" (in French), *C. R. Acad. Sci. Paris*, Feb. 21, 1949, vol. 228, pp. 651–653.

A static-pressure tube for measuring pressures over airfoils is described. It contains piezometers drilled into its 12-deg wedge end. Tests indicate that there is less angularity error than in a Prandtl tube, and that measurements can be taken accurately within 1 mm of a wall. J. M. Robertson, USA

1320. Israel Taback, "The response of pressure measuring systems to oscillating pressures," *Nat. adv. Comm. Aero. tech. Note*, no. 1819, Feb. 1949, 30 pp.

This paper presents the results of theoretical and experimental investigations of the response of pressure-measuring systems subjected to sinusoidally varying pressures at frequencies up to 70 cycles per second. The pressure-measuring systems all consist of an inlet restriction, tubing length, and a connected instrument volume. The theory, based on an analogy with the propagation of electric waves on a transmission line, is limited by the fact that no theoretical method of predicting the attenuation characteristics of the tubing is given. The author states, however, that this limitation is not severe as this characteristic may be determined experimentally.

It is shown that in long tubes of small diameter, say 20 ft of $^3/_{16}$-in. ID tubes, resonances can occur which cannot be ignored in the interpretation of recorded data. The resonance frequencies for tubes of approximately this length are of the order of those often encountered in control-surface flutter and airplane buffeting. It is concluded that, for accurate measurements of oscillating pressures, the first resonant frequency of the pressure-measuring system should be kept well above the highest pressure frequencies to be measured. John R. Spreiter, USA

1321. Morton Gertler, "Methods for the new analysis of the original data for the Taylor Standard Series," *David Taylor Model Basin Rep.*, no. 686, Feb. 1949, 39 pp.

The author presents the Taylor Standard Series used in predicting the shape, the characteristics and the effective horsepower of ships from model-test data. He describes the techniques which are used to analyze the original data for the Taylor Standard Series, the significant parameters used, and the proposed methods of presenting the data. He shows the errors due to the failure to compensate for changes in resistance caused by the variation in towing basin temperature, for transitional flow occurring at the lower Reynolds number, and for restricted channel effects.

Then he studies these different changes in resistance by reanalyzing the original test data, and the new analysis is presented as the Revised Standard Series. L. Escande, France

Thermodynamics

(See also Revs. 1299, 1300, 1311, 1316, 1317, 1328, 1339)

1322. V. P. Karpov, "Principles of engineering thermodynamics" (in Russian), Mashgiz, Moscow, 1948. Cloth, 10 × 6 in., 316 pp., 136 figs.

This textbook of thermodynamics, written for automotive engineers, closely follows in many parts Schüle's well-known presentation of the subject. The material is centered around the topics of the three la[illegible]f thermodynamics, the properties of gaseous substances, and [illegible]emical reactions. The text is well written and illuminated by well-chosen examples; thus, the book is an able introduction to the branches of thermodynamics pertinent to mechanical engineering. Among the subjects selected in the presentation of the three principal topics are, besides the standard material for texts on this subject: the relations between the partial derivatives of the various properties of state, properties of the van der Waals gas, power-cycle analysis, physicochemical changes of state, the equations of Kirchhoff, Gibbs-Helmholtz and van't Hoff, Nernst's theorem, the laws of mass action and chemical kinetics, and theories of combustion, flame propagation and detonation. Eric F. Lype, USA

1323. Joseph H. Keenan and Joseph Kaye, "Gas tables," John Wiley and Sons, New York, 1948. Cloth, 7.25 × 10.25 in., 238 pp., 5 figs., $5.

The volume constitutes an extension of the authors' earlier work *Thermodynamic properties of air* which has been revised and augmented by numerous other tables pertinent to thermodynamic and aerodynamic calculations. The new data on the thermodynamic properties in the perfect-gas region for air, nitrogen, oxygen, hydrogen, water vapor, carbon monoxide, carbon dioxide, and products of combustion mixed with air are tabulated so that they are immediately applicable to calculations of gas turbine, jet engine, and internal-combustion engine cycles. The tables of the aerodynamic properties are designed for use in subsonic as well as supersonic flow calculations, and give as functions of the Mach number the data needed for the calculation of the Fanno and Rayleigh lines and one- and two-dimensional shocks. A brief description of the tables and a few representative examples are added. Eric F. Lype, USA

1324. C. Rogers, "The derivation of variable specific heat expressions for thermal efficiency and calorific value of working fluid, for use in internal combustion engine design," *Phil. Mag.*, Feb. 1947, vol. 38, pp. 134–149.

The objective of this paper is to express the thermal efficiency of an internal-combustion engine and the heating value of the products of combustion in terms of the various design constants of the engine (compression ratio, cutoff ratio, etc.). This is done for a general power cycle which comprises the actual cycles as specific cases, when the specific heats are linear functions of temperature. No allowance is made for change in composition of the working substance during the cycle. Thus, the author obtains an efficiency formula for the general cycle containing a dozen different parameters which in turn are algebraic functions of the design constants. A similar formula is obtained for the heating value. By introducing the respective conditions for specific cycles (Otto, Diesel, etc.) considerably simpler formulas are obtained.

It is inherent in this procedure that the effect of the various design constants is not recognizable, since only combinations of these quantities appear in the equations, and nothing at all is left in these formulas of such basic thermodynamic quantities as work of

compression and expansion, heat content of exhaust gases, heat taken out by cooling, etc. which are the characteristic quantities in any power cycle. The paper will be helpful to those who find the use of entropy diagrams inconvenient, and do not mind acquainting themselves with a large number of unfamiliar parameters lacking physical meaning. Eric F. Lype, USA

1325. **J. H. Hildebrand and T. S. Gilman, "Comments on the 'Hildebrand rule,'"** *J. chem. Phys.*, May 1947, vol. 15, pp. 229–231.

According to the "Hildebrand rule," normal liquids should have the same entropy of vaporization at temperatures corresponding to equal molal vapor volumes. According to the theory of corresponding states, the entropies of vaporization should be equal for different liquids at temperatures corresponding to equal ratios of vapor volume to liquid volume.

In this paper test results with liquids having widely different liquid molal volumes are listed. It is shown that according to these results the Hildebrand rule is more closely obeyed than the second mentioned rule, which shows that the assumptions underlying the theory of corresponding states are not wholly correct.
E. Haenni, USA

1326. **Evry Schatzman, "The entropy of a gas mixture in ionization equilibrium"** (in French), *Bull. Acad. Belg. Cl. Sci.*, 1948, vol. 34, no. 10, pp. 748–753.

The adiabatic relation between pressure and temperature is important in the study of stars, due to the existence of convection zones where the heat flow may be considered to be adiabatic. The paper gives a formula for the relative specific entropy values for a gas consisting of different atoms each having several degrees of ionization. Auxiliary tables for oxygen above 1000 K are appended. R. L. Pigford, USA

1327. **Georges Claude and André-Georges Claude, "On the possibility of creating great thermo-artesian sources"** (in French), *C. R. Acad. Sci. Paris*, Apr. 4, 1949, vol. 228, pp. 1209–1210.

It is proposed to drill deep wells for the purpose of producing hot water from subterranean water layers. This hot water may then be used to operate an engine as proposed by Claude and Boucherot. H. C. Brinkman, Holland

Heat Transfer; Diffusion

(See also Revs. 1222, 1277, 1340)

1328. **L. M. K. Boelter, V. H. Cherry, H. A. Johnson, and R. C. Martinelli, "Heat transfer notes,"** University of California Press, Berkeley and Los Angeles, 1948. Paper, 11 × 8.5 in., 610 pp., 350 figs., $5.

This is a new printing of the authors' lecture notes on heat transfer which were first published in 1941. Though not intended as a textbook, the text affords a broad coverage of the various scientific and practical aspects of heat transfer. The chapters on heat conduction in solids, comprising nearly one half of the book, give a synopsis of the numerous mathematical methods and devices employed in theoretical solutions of these problems (with the exception of the Laplace transformation). Chapters on hydrodynamic concepts and dimensional analysis serve as preparation for the chapters on heat convection in which a great variety of representative problems is treated and a large number of experimental data is presented. The chapter on heat radiation, less comprehensive and with less regard to complete derivation of the formulas given, rounds out the subject. A wealth of examples illustrating the use of the various methods, and tables and charts make the volume a valuable reference book.
Eric F. Lype, USA

1329. **A. E. Benfield, "A problem of the temperature distribution in a moving medium,"** *Quart. appl. Math.*, Jan. 1949, vol. 6, pp. 439–443.

The author solves the equation of heat conduction for one-dimensional flow in a semi-infinite medium moving in the negative x direction with a constant velocity. Boundary conditions are $T(x,0) = Ax$ and $T(0,t) = -\lambda t$ where A and λ are constants. The solution is found by the Laplace-transform method which leads to a function of which the generating function is not given in the existing tables. The method used in the paper results in new useful transforms, but the problem could be solved, as indicated by the author, by other methods. This reviewer notes that the generating function of expression (7) could also be found by the use of convolution. Ahmed D. Kafadar, USA

1330. **S. S. Penner and S. Sherman, "Heat flow through composite cylinders,"** *J. chem. Phys.*, Aug. 1947, vol. 15, pp. 569–574.

The differential equations for the unsteady flow of heat through a composite cylinder are solved. The composite cylinder is assumed to be thermally insulated at the outer boundary and to consist of a cylindrical core, initially at uniform temperature U_0, surrounded by a cylindrical shell initially at temperature zero. Included are corresponding treatments for composite slabs of finite and infinite widths. Alexander Mendelson, USA

Acoustics

(See also Rev. 1222)

1331. **Philip M. Morse, "Vibration and sound,"** McGraw-Hill Book Co., New York, 1948. Cloth, 9.2 × 6.1 in., 468 pp., 92 figs., $5.50.

The first edition of this book was published in 1936 as one of the volumes of the International Series in Physics. Since its publication it has been regarded as a leading textbook in oscillation and acoustical theory. The book is divided into eight chapters. In the introductory chapter the methods of theoretical physics are discussed and a rather informal but illuminating discussion of the mathematical topics fundamental to the theory of oscillations is presented. Chapter 2 contains a very thorough discussion of the simple oscillator and of coupled oscillations. The classical electrical analogy of mechanical and electromechanical systems is presented. The impedance concept is introduced and applied to the dynamics of electromechanical systems such as piezoelectric crystals. The steady-state and transient analysis of these systems is considered.

In chapter 3, the fundamental ideas in the study of dynamical systems having distributed inertia and compliance are introduced by applying them to the analysis of the vibrations of a stretched flexible string. The next two chapters are devoted to the investigation of the vibrations of bars, stiff strings, membranes, the condenser microphone, and plates. In chapter 6 the general wave equation governing the propagation of sound in a gas is developed. The theory of the propagation of sound in tubes and horns in the steady state and the transient cases and the analogous electrical circuits is considered.

The radiation from cylinders, spheres and pistons and the scattering and absorption of sound are topics discussed in chapter 7. The design and behavior of a condenser microphone and the theory of sound transmission through ducts are also subjects considered in this chapter. In the last chapter, the mathematical methods developed for the study of the radiation of light from an atom are applied to the theory of the acoustic properties of rooms. The book is concluded with a bibliography and tables of functions throughout the text.

This volume is an excellent textbook and reference work. In the discussion, the fundamental physical ideas are emphasized, and often questions of generality and mathematical rigor have been sacrificed to make the chain of logic more distinct. The author is very skillful in replacing rigid technical vocabulary by colloquial phrases in order to make concepts vivid. As an example of this, the concept of steady-state motion is explained in the following manner: "Steady-state motion is motion of a system that has forgotten how it started." The second edition of this book is more than 100 pages longer than the first edition. Most of the new material is the analysis of the transient behavior of the various oscillatory systems considered. The transient analysis is effected by the use of Fourier and Laplace transform methods. These methods make use of integration in the complex plane and the theory of residues. There is a collection of interesting problems at the end of each chapter.

The notation used differs from the notation of the electrical engineers in that it is necessary to replace every i in the formulas of the book by $-j$ to obtain the usual electrical-engineering expressions. Although frequent use is made of electrical analogies of the various dynamical systems considered, the use of the circuit analogy is not carried out to the fullest possible extent as is the current fashion in the study of vibrating systems. It is unfortunate that no mention is made of four terminal structures or the iterative matrix methods used in determining the natural frequencies and modes of dynamical systems. The conventional notation of the Laplace transform theory is not used. The value of the book as a reference work would be increased if a more detailed bibliography were included. Louis A. Pipes, USA

Ballistics, Detonics (Explosions)

(See Rev. 1286)

Soil Mechanics, Seepage

1332. Henri Labasse, "Soil pressure in coal mines, I" (in French), *Rev. univ. Min.*, Jan. 1949, vol. 5, pp. 3–15.

In compression tests on rock three cases may be distinguished: (*a*) simple compression, applied to two opposite faces of a cubical sample; (*b*) partial confinement, when compression acts on four faces, and (*c*) complete confinement, when all faces are acted upon by compression. In case (*a*), and in case (*b*) when the bearing plates are greased to prevent friction, the sample fails by falling apart into a number of vertical prisms. Otherwise in case (*b*) failure planes occur at angles of 45 deg. Case (*c*) was studied by von Kármán on marbles and sandstones some 25 years ago. Large deformations in von Kármán's tests, which the author identifies with a plastic state of the sample, occurred at lateral pressures in the triaxial test of the order of 275 atm, which (assuming the Poisson ratio of rock to be 0.2) corresponds to a depth over 13,000 ft. Since large deformations in mines occur at smaller depths than this, the author believes that the real cause of such deformations is not the passing of the matter into a plastic state, but the fissuring of the rock material during the excavation ("exploitation" of the mine). A fissured bank keeps its integrity or geometric continuity while being discontinuous physically. According to the author, the material around a hole or an excavation in rock behaves as if "the matter, formerly solid and fragile, becomes plastic and tends to fill up the empty space." The author concludes that rock is a semielastic material. Under the action of a suddenly applied force, such rock material does not exhibit any plasticity and in this case it is a nonplastic material. It is not elastic either, but may become elastic after being submitted to a certain number of loading-unloading cycles in which a given maximum compression stress is reached.

The Rankine formula is discussed, as well as application of the Mohr circle and Mohr envelope to rocks. Data referring to the weight of some rocks and soils, their resistance to compression and shear, their angle of internal friction and Poisson's ratio are given. Reference is also made to Fenner's work on rock pressures [Gluckauf, 1938]. D. P. Krynine, USA

1333. H. Labasse, "Soil pressure in coal mines, II" (in French), *Rev. univ. Min.*, Mar. 1949, vol. 5, pp. 78–88.

This second paper by Labasse (see preceding review) concerns pressure in rocks. Its final objective is the determination of the pressure on the lining of a vertical shaft. In the case of rocks of high compressive and shearing strength, the lining of the shaft is placed only as a matter of precaution against accidentally falling rock fragments; stresses around the shaft may be computed by simply using the theory of elasticity, and neglecting the effect of the lining. In the case of rocks that are fractured or may be fractured, the sinking of a shaft causes fragmentation of the surrounding rock by concentric zones. The deeper this propagation, the smaller the pressure on the lining. Hence placing of a rather rigid permanent lining immediately after the sinking of a shaft is not advisable, since in this case the propagation of the fragmentation is prevented, and the lining has to take up the maximum possible pressure. Instead, work by gradual strokes ("passes") using temporary flexible lining is recommended.

The actual pressure on a shaft lining is somewhere between the minimum pressure as furnished by the theory of elasticity, and the maximum pressure computed for the case of plastic equilibrium using the Rankine ratio of principal stresses. The choice of the proper value is a matter of judgment.

The paper is illustrated by various diagrams, and accompanied by derivation of formulas and numerical examples. Particularly the author's stress-distribution diagram in the plastic and elastic zones (Fig. 21) is similar to those describing stresses around a shaft in sand [for inst., Fig. 61 of Terzaghi, *Theoretical Soil Mechanics*, New York, 1943], without being completely identical, however. The author, besides circular shafts, discusses also elliptical and rectangular shafts. D. P. Krynine, USA

1334. L. A. Palmer and James B. Thompson, "Pavement evaluation by loading tests at naval and marine corps air stations," *Proc. Highw. Res. Bd.*, 27th Ann. Meet., 1947, pp. 125–143.

The U. S. Navy Department found it useful to investigate the behavior of existing airfields with regard to pavement and subgrade conditions. For this purpose a direct study was made of 54 airfields and the results are presented in the present article.

Most of these pavements were of the flexible type 6 to 12 in. thick, and had been in service for 3 to 6 years, sustaining an intense traffic, with wheel loads ranging from 2500 to 60,000 lb for a wheel. Tests were made by direct loading of the pavement and a settlement of 2 in. was considered as an allowable deformation. At the same time the geotechnical characteristics of the subgrade were established by means of field laboratory analysis.

In this article the results for 32 airfields are given in tables and graphs and are amply discussed. The conclusion is that subgrade analysis is of great importance for pavement design. With regard to subgrade moisture and density, experience has proved that especially flexible pavements consolidate the subgrade by reducing the moisture and increasing the density.

Aurel A. Beleş, Rumania

1335. A. Schleusner, "On a problem in the theory of earth pressure," *Reissner anniv. Vol.*, J. W. Edwards, Ann Arbor, 1949, pp. 248–255.

The author presents a mathematical solution of the problem of determining the limits of the force on a piston closing an opening in the bottom of a container filled with sand. Assuming the stress to vary radially, integral expressions for the maximum and minimum pressures are determined for a circular piston and for a long rectangular piston. For values of the coefficient of internal friction greater than 19.5 deg, the expression for minimum pressure is integrated, yielding an infinite series with improved convergence. No numerical evaluation is given. Experimental corroboration of the theory by experiments with Goldbeck pressure gages is to be reported elsewhere.

Edward S. Barber, USA

Geophysics, Meteorology, Oceanography

1336. G. P. Cressman, "On the forecasting of long waves in the upper westerlies," *J. Met.*, Apr. 1948, vol. 5, pp. 44–57.

This article presents results of various statistical and synoptic tests of relationships predicted by Rossby's theories concerning waves in the westerlies. It is found that, using the average zonal speed computed from the 600-mb constant-pressure map, the nondivergent wave formula gives accurately the speed of waves in broad westerly currents about 70 per cent of the time. For progressive waves in narrow "jet streams" the wave movement corresponds to that predicted by comparison of the wave length with that for the corresponding constant vorticity trajectory. Retrogression of major waves consists in a weakening of original trough, and a development of a trough farther west. Such retrogression occurs successively on all troughs downstream of the original retrograde trough.

Changes in the shape of waves generally occur in connection with the meridional shift of the zone of maximum westerlies, or a change in wave length. Changes in amplitude are associated with the progression of energy downstream from one wave to the next at the group velocity.

No measures of variability or tests of significance of the means are presented in connection with the statistical results.

M. Neiburger, USA

1337. George P. Cressman, "Some effects of wave-length variations of the long waves in the upper westerlies," *J. Met.*, Feb. 1949, vol. 6, pp. 56–60.

The theoretical models in meteorology are simple and restricted in comparison to the vast complexities of the real phenomena. A carefully controlled test is extremely difficult to make, and the present paper pioneers in bridging the gap between meteorological theory and practice.

The advances made by Rossby [C. G. Rossby, "On the propagation of frequencies and energy in certain types of oceanic and atmospheric waves," *J. Met.*, 1945, vol. 2, pp. 178–203] and Yeh [T. C. Yeh, "On energy dispersion in the atmosphere," *J. Met.*, 1949, vol. 6, pp. 1–16] in the study of the dispersive nature of atmospheric waves are applied to forecasting the long-wave patterns in the westerlies 72 hours in advance. The author's analysis shows that the acceleration of the waves depends on the change of wave length upstream, the waves speeding up when the wave length decreases upstream. The equations are evolved and set up in units immediately suitable for application to weather maps. Graphs are provided by which all necessary computations can be easily made.

Results of the author's tests show that application of the concept of dispersion improved the verification of the forecast in 38 out of 50 cases, and the 72-hour position of the wave is obtained within 10 deg long. in nearly all cases. As in his previous work [G. P. Cressman, "On the forecasting of long waves in the upper westerlies," *J. Met.*, 1948, vol. 5, pp. 44–57], the author chooses the cases to be studied, so as to most nearly conform to the conditions under which the theory was derived.

Joanne Starr Malkus, USA

1338. Dave Fultz, "A preliminary report on experiments with thermally produced lateral mixing in a rotating hemispherical shell of liquid," *J. Met.*, Feb. 1949, vol. 6, pp. 17–33.

The experimental equipment used by the author consists of two concentrically mounted hemispheres which are rotated about the polar axis. The region between the hemispheres is filled with fluid which is heated at the pole (at the bottom of the hemisphere). Zonal motions of the fluid are traced by observing, either visually or photographically, the motions of dye streamers or of suspended pellets. With rotation but no heating, or with heating but no rotation, no zonal motions of the fluid with respect to the hemispheres were noted. With both heating and rotation, strong relative zonal motions were observed which have a curious resemblance to the mean zonal motions in the earth's atmosphere. For example, "easterly" currents were observed near the equator, "westerly" currents were observed in the middle latitudes, and "anticyclonic" eddies were observed in the region of and between these two zones. The author attempts to interpret the flow as an atmospheric model.

No attempt, beyond an elementary dimensional analysis, is made to investigate the dynamical equivalence of the model and the earth's atmosphere. It should be noted that there are at least two important differences. First, the heat-flux vector in the model is from the pole toward the equator. Second, the resultant acceleration field (gravity plus rotation) in the model has a strong component tangential to the hemispherical surfaces in the direction of the pole, except in the immediate neighborhood of the pole. Thus the differential heating produces an unstable stratification of the fluid which must be expected to induce directly meridional motions.

The experimental results are extremely interesting; however, it is the opinion of the reviewer that a much more complete dynamical analysis of the model would have to be carried out before a significant analogy with the earth's atmospheric motions might be deduced.

H. J. Stewart, USA

1339. D. Fraser, "Induced precipitation—preliminary experiments on seeding a variety of clouds with dry ice," *Nat. Res. Counc. Canada Mech. Engng. Rep.*, no. MR-4, Jan. 18, 1949, 15 pp.

In view of the bitter controversy raging over dry-ice seeding, any objective evidence is welcome. This paper, however, cannot be regarded in any way as conclusive due to difficulties in controlling the experiments and to scanty instrumentation. It merely indicates that dry-ice seeding frequently has an observable effect on clouds, but the question as to how the particular effect can be controlled or predicted is not answered.

Joanne Starr Malkus, USA

1340. L. M. K. Boelter, H. Poppendiek, G. Young, and J. R. Andersen, "An investigation of aircraft heaters, XXX—Nocturnal irradiation as a function of altitude and its use in determination of heat requirements of aircraft," *Nat. adv. Comm. Aero. tech. Note*, no. 1454, Jan. 1949, pp. 1–56.

Calculations of the nocturnal irradiation of an aircraft are made on the assumption that water vapor, carbon dioxide, and ozone are the only important radiating components, apart from the earth itself. Results are presented in the form of charts from which the irradiation of a horizontal surface from below and above can be obtained. An example is given which shows that this heat radiation is not insignificant in the problem of aircraft heating in conditions of high humidity. R. Smelt, USA

1341. J. A. Putnam and J. W. Johnson, "The dissipation of wave energy by bottom friction," *Trans. Amer. geophys. Un.*, Feb. 1949, vol. 30, pp. 67–74.

It has long been known that the effect of molecular viscosity on waves in deep water is very small. Thus for 10-second waves the modulus of decay is 6 years. This has led to the erroneous assumption that friction need not be considered in any of the phases of wave motion. The authors have demonstrated that as a result of friction by the oscillating motion of waves along the sea bottom the wave energy may be reduced by as much as 30 per cent before the waves break. Methods for predicting wave characteristics over very flat beaches should be modified in the light of this investigation. Walter H. Munk, USA

Lubrication; Bearings; Wear

1342. D. Downs and J. H. Pigneguy, "Lubricating oil tests," *Auto. Engr.*, Feb. 1949, vol. 39, pp. 51–58.

A series of accelerated engine tests has been evolved for rating oils, especially lighter duty oils containing antioxidants, detergents and dispersants. The tests are made on water-cooled gasoline engines. Piston cleanliness tests last 50 hr, the temperature of the piston being 200 C and that of the oil 95 C; ratings are given depending on the extent of carbon deposit on the skirt. Cold-sludging tests last 200 hr, the temperature of the jacket and the oil being lower than in the previous test. Big-end bearing corrosion tests last 90 hr; copper-lead bearings are used and the oil temperature is maintained at 100 C. Cylinder and ring wear is tested in a special crosshead engine providing separate lubricating oil for the cylinder and the crankcase assembly. Ring sticking tests are carried out for standard periods of 5 hr, the cylinder temperature increments between successive tests being 5 C until a quarter of the top ring is tight in the groove.

Diagrams and photographs of pistons show that oil performance can be essentially improved by some additives.

Ferd. Budinský, Czechoslovakia

1343. M. C. Shaw and T. J. Nussdorfer, Jr., "An analysis of the full-floating journal bearing," *Nat. adv. Comm. Aero. Rep.*, no. 866, 1947 (issued in 1949).

The authors demonstrate by analysis that full-floating journal bearings, that is, bearings in which a sleeve is free to rotate between the journal and bearing proper, operate at lower temperatures than conventional bearings because less total heat is generated in the two oil films, and there is an over-all increase in oil flow. Notwithstanding the fact that their analysis follows Sommerfeld in that the film is presumed to be continuous completely around the bearing, experiments in which the ratio of outer to inner clearance is 0.43 show excellent agreement with theory; but where this ratio is unity, agreement is less good at the lower values of Sommerfeld number (below 30). When the ratio was 1.67 the bearing was unstable in operation.

The speed of rotation of the sleeve is shown to be dependent on the ratio of inner to outer bearing clearance and the results are confirmed by experiment. The main disadvantage of this type of bearing appears to be the risk that the sleeve will not rotate. This trouble is unlikely to be met with where load varies, or where vibrations are present. The paper concludes with the discussion of bearings in which the sleeve is pegged either to the shaft or the bearing and the authors show that the heat generated in a floating bearing of this type is always greater than that developed in the conventional journal bearing. F. T. Barwell, Scotland

1344. Warren G. Payne and William F. Joachim, "Investigations on cylinder-liner wear," *Soc. auto. Engrs. quart. Trans.*, Jan. 1949, vol. 3, pp. 51–68.

A group of 24 Diesel 2- and 4-cycle engines covering a wide range of design and operating characteristics were selected for investigation of the causes of cylinder-liner wear. The engine factors which were found to have a pronounced influence on cylinder wear included bore size, piston speed, cylinder displacement, bmep, liner hardness, piston-skirt clearance, fuel consumption, lubricating-oil consumption, lubricating-oil temperature and cooling-water temperature. These factors were investigated on both plain cast-iron cylinder liners and on chromium-plated cylinder liners. In general, it was found that the chromium-plated cylinder liners had about one-quarter the average wear of the unplated cylinder liners.

A type of wear is indicated on these liners which is not described by the present theory of wear. This wear appears to be caused by general surface disintegration which results in a loss of material in the form of very fine particles. It would seem that more work is required to find the basic reasons for this kind of wear and its mechanism before too much improvement can be made to counteract it. Erle I. Shobert II, USA

Marine Engineering Problems

(See Revs. 1288, 1317, 1318, 1321)